Introduction to Astrophysics

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Outline

Astrophysics, Detectors and Astronomical objects

What is Astrophysics ?

- Astrophysics is the study of the Universe and its components

- It evolved from astronomy towards astrophysics in the 1940-50's thanks to the discovery of
  - nuclear fusion of hydrogen as the source powering stars (Arthur Eddington 1920, Cecilia Payne 1925)
    This is remarkable as, at that time, nuclear fusion and the fact that stars are composed mostly of hydrogen was not yet discovered.

  - the expansion of the universe (Edwin Hubble 1929).

What is Astrophyics ?

Before, it was mostly an observational subject whose main driver was to classify celestial objects or to measure their position and compute their motion.

The first breakthrough however can be associated to the discovery of spectral lines in the Sun's spectrum by William Hyde Wollaston and Joseph von Fraunhofer in the second part of the 18th century.

Astrophysics is a large Field

Topics in Astrophysics include

- cosmology
- planetary sciences
- exobiology
- stellar physics
- asterosismology
- interstellar medium
- astrophysical plasmas
- high energy astrophysics
- instrumentation
- galaxies
- astroparticle physics
- ...

Astrophysics is a large Field

The subject being very broad, astrophysicists apply concepts and methods from various fields of physics such as

- classical mechanics
- electromagnetism
- statistical mechanics
- thermodynamics
- quantum mechanics
- relativity,
- nuclear and particle physics
- atomic and molecular physics
- ...

Astronomical detectors

Type of detectors

We use different type of detectors, some are ground-based, some are space based with different means.

- optical imaging (photometry)
  - interferometric detectors
- spectroscopy
- radio detectors
  - single dish
  - interferometers

At the interface with particle physics (astroparticles)

- Cherenkov arrays
- Gravitational wave detectors (LIGO/VIRGO)

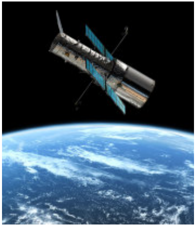
Example of optical/NIR telescopes

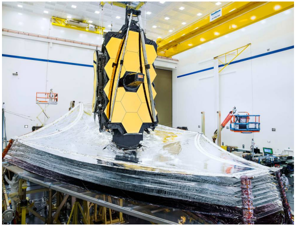
CFHT in Hawaii

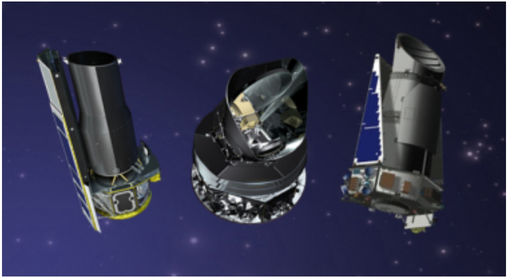
LBT in Arizona

Example of optical/NIR telescopes

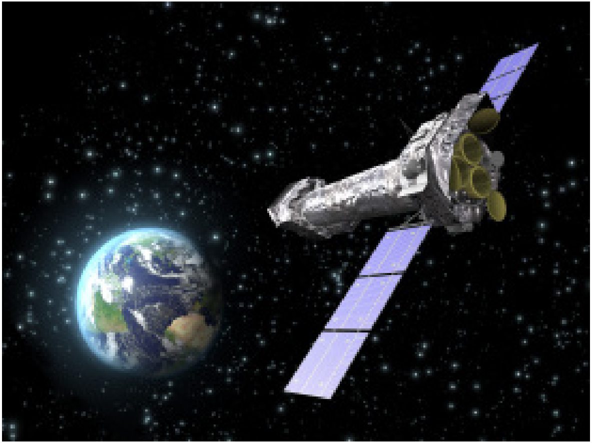
Example of space telescopes : HST, JWST,Spitzer, Planck, Kepler







Example of space telescopes : XMM-Newton



Example of single dish radio-telescopes : Arecibo, Effelsberg

Arecibo

Effelsberg 100m

Example of single dish radio-telescopes : Onsala installations

Example of radio/SMM inteferometers : ALMA

Example of radio/SMM inteferometers : VLA

Example of radio/SMM inteferometers : SKA

Physical processes and measurements associated to the electromagnetic spectrum

- radio (3MHz – 30 GHz / 100m – 1cm)
  - thermal bremsstrahlung from ionized hydrogen
  - neutral hydrogen (21 cm line)
  - synchrotron radiation from relativistic plasma (interstellar magnetic field, radio lobes of active galaxies, quasars, pulsars...)
  - precise parallaxes (interferometry)
- (sub-)millimeter (30 GHz – 3 THz /10 – 0.1 mm)
  - thermal bremsstrahlung from ionized hydrogen
  - observation of molecular lines
  - Cosmic Microwave Background

Physical processes and measurements associated to the electromagnetic spectrum

- infrared (3 THz – 30 THz / 0.1 mm – 1 μm)
  - thermal emission from stars, galaxies, AGN...
  - emission from dust grains
- optical (30 THz – 1 PHz / 1 μm – 300 nm)
  - thermal radiation from planets, stars, galaxies ...
  - emission and absorption features
  - astrometry (position and proper motion, parallaxes)
- ultraviolet (1 PHz – 30 PHz / 300 – 10 nm)
  - thermal radiation, non-thermal radiation from active galaxies and quasars
  - resonance lines of ions, atoms

Physical processes and measurements associated to the electromagnetic spectrum

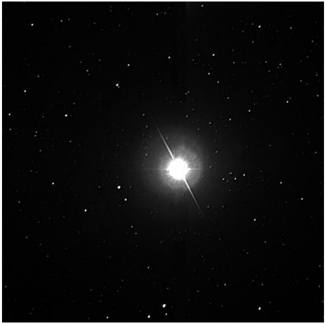
- X-ray (30 PHz – 30 EHz / 10 nm – 10 fm)
  - supernova remnants, AGN, pulsars, binaries
  - thermal bremsstrahlung in galaxy clusters
- γ-ray (> 30 EHz / < 10 fm / E > 100 keV)
  - non-thermal processes (bremsstrahlung, inverse Compton, decay of pions...)
  - radioactive elements (line emission)
  - e^+e^- annihilation line (511 keV)
  - SN remnants, AGN, pulsars, γ-ray bursts
- gravitational waves signal
  - physics of coalescence, equation of state of degenerate matter
  - general relativity

Overview of astronomical objects

Simple (single) Objects

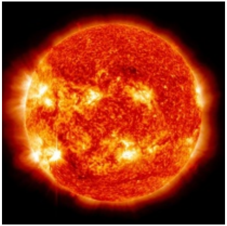
Stars can be thought off as simple objects but they are not so simple. They show a diversity in

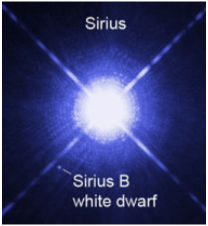
- mass
- age or evolutionary stage
- chemical composition
- ...



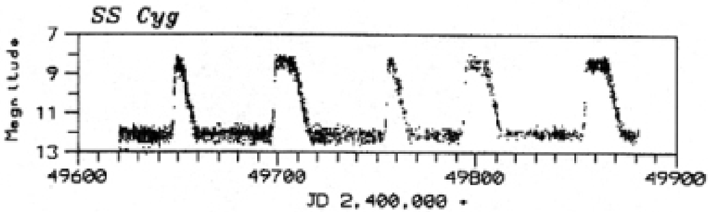
Stars

These simple objects can be quite complex (activity, winds, companions...)

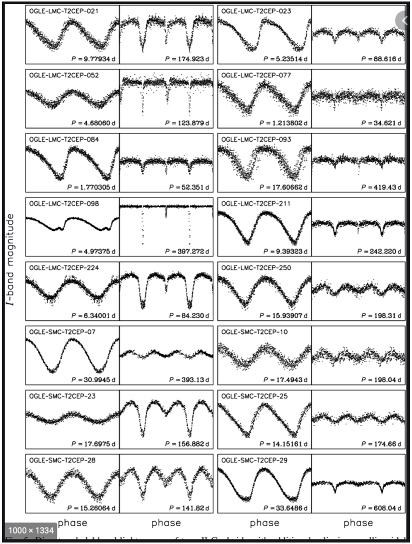




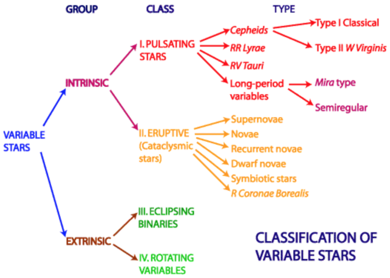
and show variability in their photometric properties such as dwarf nova



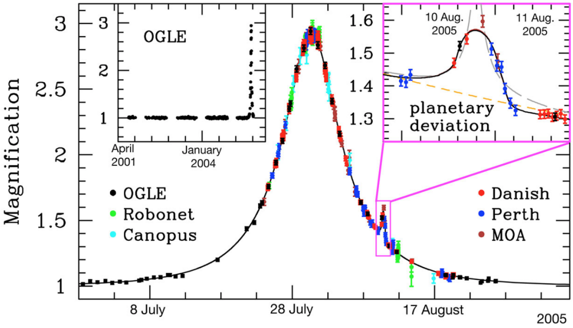
or other type of variable stars



There exist a wide variety of variable stars

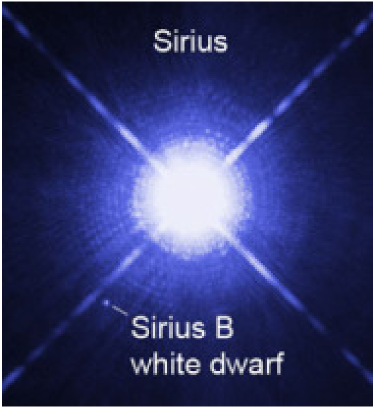


Other type of photometric behaviour : faint stars (low mass) can be detected via microlensing effect of a background star such as in this example

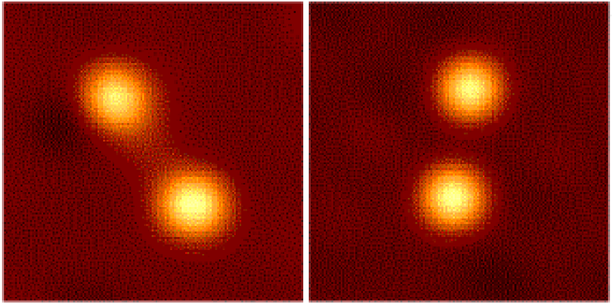


Multiple systems

There exist also multiple stellar systems for example binary stars



Multiple systems



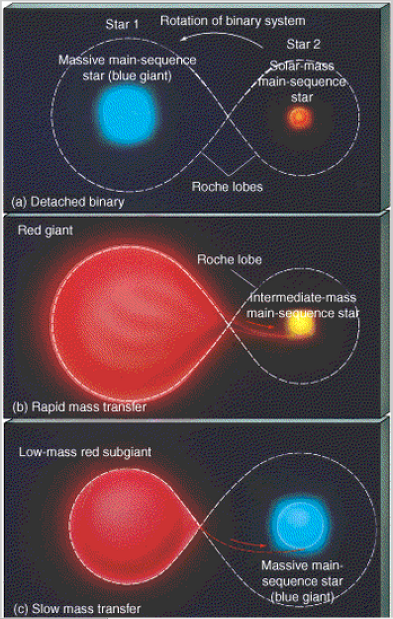
Images of Capella taken on the 13th (left) and 28th (right) September 1995. The separation between the stars is 55 milli-arcsec.

Multiple systems

It can be more than just two stars, systems with up to five stars have been observed :

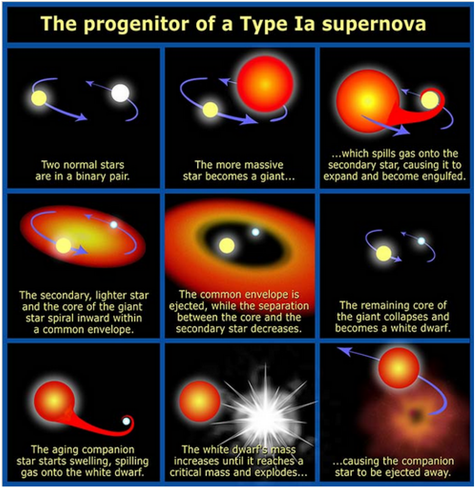
- great variety of binary systems with large range of orbital periods
- the more massive component is named the primary, then secondary...
- mass transfer between the stars (secondary to primary) via Roche lobe overflow
  - high energy processes happen when matter flows onto an accretion disk

Multiple systems



Multiple systems

Roche lobe overflow can even lead to the explosion of the system



On larger scale, we find star clusters such as open clusters

Stars in open clusters are not bound by gravity. Their lifetime does not exceed a few rotation around the galaxy.

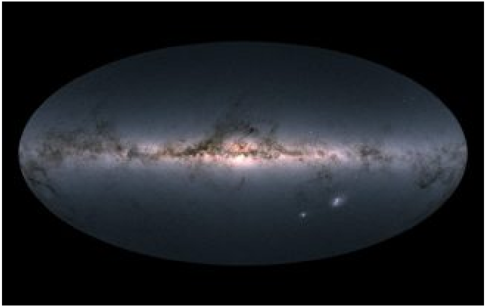
Multiple systems

More massive than the open clusters, we have globular clusters :

They are also older. Stars in those systems are gravitationally bound.

Interstellar medium

The space between stars is not empty. It is filled by what we call the interstellar medium



Dark regions are in fact regions where the light is absorbed by materials in the Galaxy.

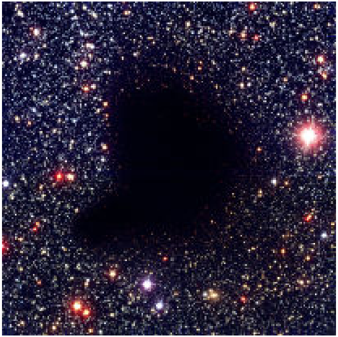
Interstellar medium

It is also seen in other galaxies such as the Sombrero Galaxy

These dust lanes are the proof that matter exists between stars.

Interstellar medium

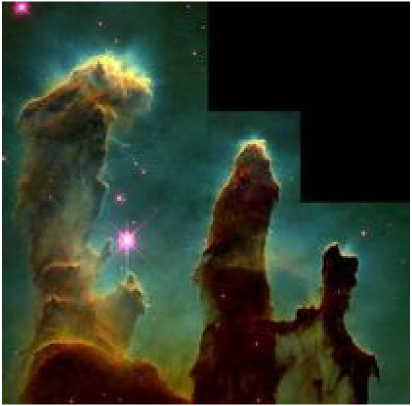
Here is a close up on a small region of the sky. The empty region is an interstellar cloud whose density is high enough to block the light coming from sources behind (Bok nodules).



Interstellar medium

In fact, the interstellar medium is a complex structure.

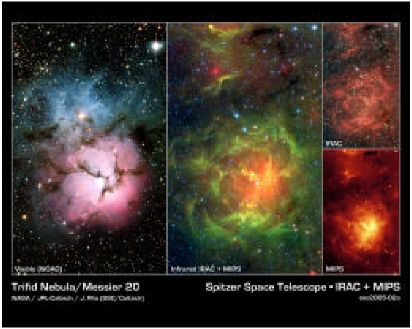
Interstellar medium



It is also the site of star formation and new stars are formed from material inside molecular clouds.

Interstellar medium

The interstellar gas can be ionised by the surrounding radiation

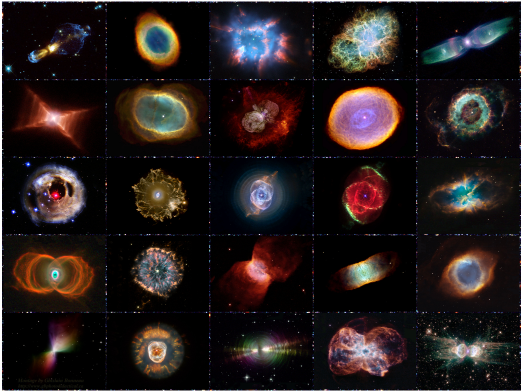


Planetary nebula

At the interface between stars and interstellar medium we find planetary nebula - PN - such as M57 (the ring nebula). Planetary nebula are the late stage of stellar evolution where the star expels its atmosphere in the surrounding medium while the core of the stars collapses to form a white dwarf.

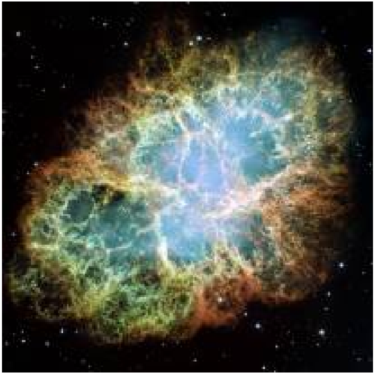
Planetary nebula

They come in various sizes and shapes depending on the age and other parameters of the parent star



Supernova

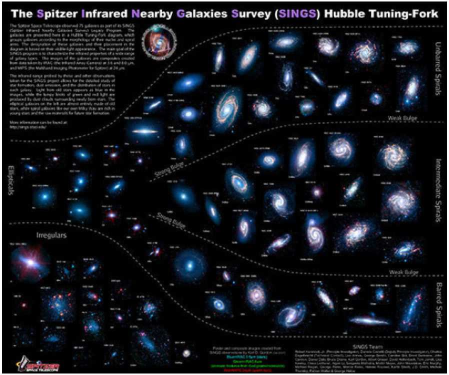
More massive stars explode to form supernovae (SN) such as the crab nebula



On larger scale, we find galaxies such as the Milky Way or Andromeda (M31)

Galaxies

Galaxies are of different sizes and shape but their morphological shape can be classified using for example Hubble's fork

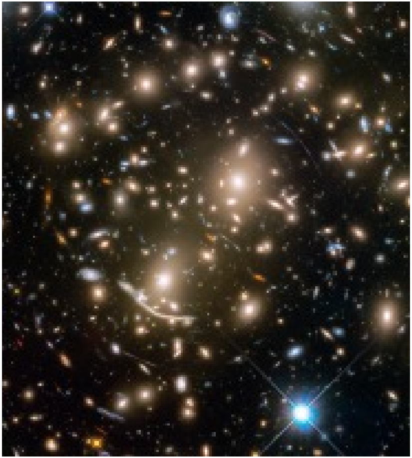


Groups and clusters

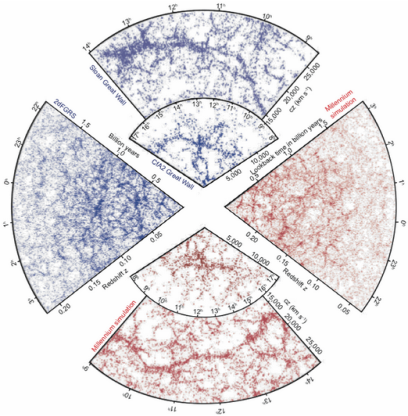
Galaxies can be isolated but they are found most commonly in groups or clusters !

Groups and clusters

Clusters are more massive and contain more galaxies than groups.



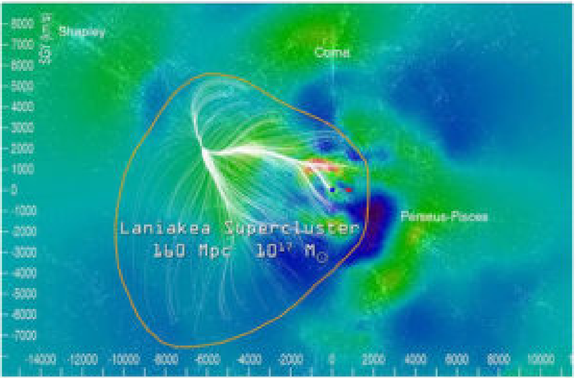
Cosmic structures



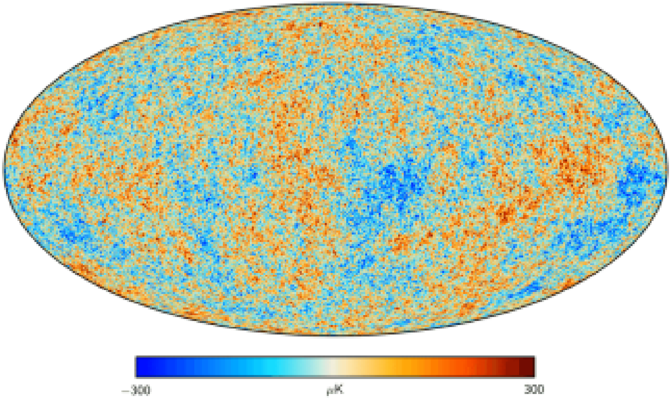
Galaxies and cluster arrange themselves in filaments and sheets via gravity and expansion of the universe

Superclusters

and superclusters of clusters of galaxies and galaxies do exist



The "largest" cosmic structure : the cosmic microwave background (CMB)



Motion of the Earth

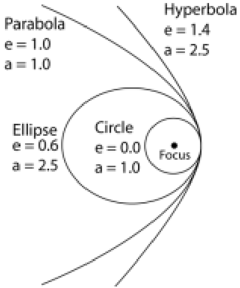
Conic sections

Conic sections : definitions

Let F be a point of the plane, D a line not including F. Let e be a real such as $e > 0$. The conic of focal F, directrix D and eccentricity e is the set of points M of the plane such that $MF = eMD$.

Three different cases :

- $0 < e < 1$: ellipses
- $e = 1$: parabola
- $e > 1$: hyperbola



Conic sections : cartesian coordinates

Given the coordinate system $(F, \hat{i}, \hat{j})$ such that the directrix has equation $x = d$.

The cartesian representation of the conic of focus F, directrix D and eccentricity e is given by

$$x^2 + y^2 = e^2.(x - d)^2$$

Conic sections : cartesian coordinates

ellipses : There exist a coordinate system where the cartesian equation of an ellipse writes $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with $a > b > 0$. The Focus has coordinates $F(c,0)$ with $c = \sqrt{a^2 - b^2}$. Eccentricity is $e = \frac{c}{a}$

parabola : $e = 1$. The equation of the parabola is $y^2 - 2dx = d^2$

hyperbola : $e > 1$. Equation $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

admits two asymptotes : $y = \frac{b}{a}x$ and $y = -\frac{b}{a}x$

Conic sections : polar coordinates

Polar equation

$$\rho = \frac{p}{1 + e\cos(\theta)}$$

p is the semi-latus rectum.

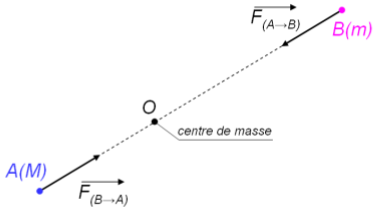
If the directrix writes $x = d$, then $p = e.d$

Two body problem

Statement of the problem

Given two point masses A and B of masses M, respectively m. Let O be the center of mass of the system. We define $\vec{r} = \vec{r}(t) = \overrightarrow{AB}(t)$

Using Newton's law : $\overrightarrow{F_{A\to B}} = -\overrightarrow{F_{B\to A}} = -\frac{GMm}{r^3}\vec{r}$



Two body problem

If the system does not experience any external force, then the center of mass is either stationary or in a uniform (linear) motion with respect to a Galilean reference frame.

- O can be considered a fixed point. A and B are in rotation around that point.
- The problem can be simplified

Two body problem

By definition of the center of mass we have

$$\overrightarrow{OA} = -\frac{m}{M+m}\overrightarrow{r} \quad \text{and Â} \quad \overrightarrow{OB} = \frac{M}{M+m}\overrightarrow{r}$$

Considering B, Newton second law states :

$$\overrightarrow{F_{A\to B}} = m\frac{\mathrm{d}^2\overrightarrow{OB}}{\mathrm{d}t^2}$$

Combining the previous equations we obtain

$$-\frac{GMm}{r^3}\overrightarrow{r} = \frac{Mm}{M+m}\frac{\mathrm{d}^2\overrightarrow{r}}{\mathrm{d}t^2}$$

Introducing the reduced mass $\eta = \frac{Mm}{M+m}$ the problem rewrites

$$-\frac{GMm}{r^3}\overrightarrow{r} = \eta\frac{\mathrm{d}^2\overrightarrow{r}}{\mathrm{d}t^2}$$

which is the equation of motion of a point C of mass η such that $OC = r$ in a variable central force field.

Two body problem

The rate of change of the angular momentum is given by

$$\begin{aligned}\frac{\mathrm{d}\overrightarrow{L}}{\mathrm{d}t} &= \frac{\mathrm{d}}{\mathrm{d}t}\left(\eta\overrightarrow{r}\wedge\frac{\mathrm{d}\overrightarrow{r}}{\mathrm{d}t}\right)\\ &= \eta\left(\frac{\mathrm{d}\overrightarrow{r}}{\mathrm{d}t}\wedge\frac{\mathrm{d}\overrightarrow{r}}{\mathrm{d}t}+\overrightarrow{r}\wedge\frac{\mathrm{d}^2\overrightarrow{r}}{\mathrm{d}t^2}\right)\\ &= \overrightarrow{0}\end{aligned}$$

The angular momentum is constant in both its orientation and norm.

Two body problem

Can be rewritten $\overrightarrow{L} = \eta C \overrightarrow{z}$

In polar coordinates :

$$\overrightarrow{L} = \eta r^2 \dot{\theta} \overrightarrow{z}$$

which leads to $r^2\dot{\theta} = C$ which is Kepler's second law as $\frac{\mathrm{d}S}{\mathrm{d}t} = \frac{1}{2} r^2 \dot{\theta}$.

Two body problem : Binnet's formula

(velocity and acceleration in polar coordinates)

Define $u := \frac{1}{r}$, $\dot{x} := \frac{\mathrm{d}x}{\mathrm{d}t}$, $x' := \frac{\mathrm{d}x}{\mathrm{d}\theta}$ and $C = r^2\dot{\theta}$.

Then

$$\overrightarrow{v} = -C\,u'\,\overrightarrow{e_r} + C\,u\,\overrightarrow{e_\theta}$$

and

$$\overrightarrow{a} = -C^2\,u^2\,(u'' + u)\,\overrightarrow{e_r}$$

Two body problem : Binnet's formula

Using the definition of angular momentum + the formula above we obtain

$$\frac{\mathrm{d}^2 u}{\mathrm{d}\theta^2} + u = \frac{G(M+m)}{C^2}$$

$\Rightarrow$ equation of an harmonic oscillator.

Solution is given by

$$u = \frac{GMm}{\eta C^2} + \alpha \cos(\theta - \theta_0)$$

or

$$r = \frac{\frac{C^2}{G(M+m)}}{1 + \frac{\alpha C^2}{G(M+m)} \cos(\theta - \theta_0)} = \frac{p}{1 + e \cos(\theta - \theta_0)}$$

$\Rightarrow$ Trajectories of A and B are conic sections

Motion of the Earth

Motion of the earth

Why do we need to know it ?

The knowledge of the motion of the Earth is fundamental

- one must be able to locate an object on the celestial sphere
- the observer follows the motion of the Earth, hence this motion modifies the apparent motion of an object
- complex motion : about twelve different components

Current observations and models able to predict this motion to $\sim 10\,\mathrm{cm/s}$

Intrinsic motion

Primary component : Earth rotation

- 1 rotation in 23h56m4.09s (sidereal day defined by two subsequent passages of a given star at the observer's meridian)
- rotation from east to west around the axis of the poles (direct rotation)
- linear velocity of $v_o = 465.1\text{km/s}$ at the equator
- $v = v_o \cos\phi$ at the latitude ϕ

Intrinsic motion

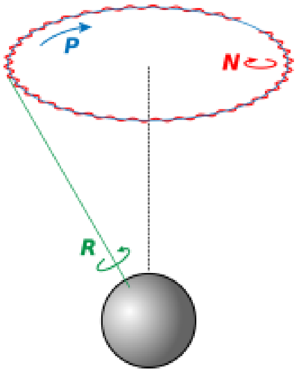
Precession of the polar axis

- conic motion of the polar axis around its mean pole
- due to the torque generated by the Sun and the Moon which tends to bring the equator in the ecliptic plane (plane of the orbit of the Earth around the Sun)
- cone of opening $23°26'19.34"$ (obliquity of the ecliptic)
- period of precession (rotation) 25 770 years
- North pole is not fixed in time
  - Intersept of the equator and the ecliptic (equinoxes) not fixed in time.
  - Backwards motion of 50.3"/year ⇒ precession of the equinoxes

Nutation of the polar axis

- oscillatory motion around the mean pole
- more than just one nutation observed
- most important : Bradley's nutation
  - due to the Moon whose orbit is inclined with respect to the equator
  - elliptic motion of axis 18.4" around the mean pole
  - period of 18.6 years

Intrinsic motion



Intrinsic motion

In addition to the regular motion, motions due to irregularities of the earth's density, atmospheric and oceanic current, tides, perturbation of solar system objects ... also contribute.

⇒ they influence all the components of the earth motion including its rotation

The Earth is not a perfect sphere : flattening of the poles

Currently, the position of the earth axis is known to 0.1 mas

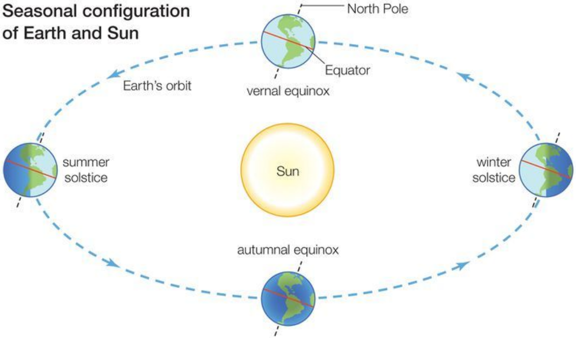
Motion of the Earth with respect to the Sun

The Earth follows an elliptic orbit around the Sun

- its plane is called the ecliptic plane
- small eccentricity $e = 0.0167$ (e being the ratio of the focal distance to the semi major axis)
- full rotation in 365d 6h 9m 9.75s (direct rotation)
- perihelion and aphelion are the closest and farthest point to the Sun on the orbit
- equinoxes Υ (march) and Υ' (automn) when the axis Sun-Earth is contained in the equatorial plane
- solstices (summer and winter) : the apparent height of the Sun is extremal

Υ is called the vernal point

External motion



Kepler's laws :

1st : **law of orbits**. Trajectories are elliptical with the Sun at one of the two foci

$$r = a\frac{1-e^2}{1+e\cos v}$$

with a the semi-major axis, e the eccentricity, v the angle between the perihelion, the Sun and the earth and r the Sun-Earth distance

External motion : Kepler's law

2nd : **law of equal area**. The area swept on the orbit for a fixed time interval is constant.

$$\frac{1}{2} r^2 \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\pi a^2 \sqrt{1-e^2}}{P} = \mathrm{cste}$$

(e.g. areal velocity is a constant)

External motion : Kepler's law

3rd : **law of periods**

$$\left(\frac{2\pi}{P}\right)^2 a^3 = G(M_\odot + m)$$

if P in yr, a in AU and masses in $M_\odot$:

$$P = a^{\frac{3}{2}}$$

Motion of the Earth in the Galaxy

- The barycentre of the solar system is not fixed in space. It orbits the Galaxy and moves toward a direction called the apex with a velocity of 19.5 km/s with respect to the neighboring stars
- The circular rotation velocity around the Galactic center is 220 km/s (V_{co} @ 8kpc)
  - a complete rotation around the Milky Way takes 250 Myr

Moon and tides

Moon and tides

Simplified description neglecting the influence of the Sun : the moon orbits the earth on an elliptical orbit around the gravitational center of the earth-moon system

- The plane of this orbit is inclined by 5° 8' 43" on the ecliptic plane (small compared to the inclination of the equatorial plane), $e = 0.0549$
- synodic month : between two conjunction of the moon and the sun (2 NMs), length 29d 12h 44' of TU
  - in 1h the moon moves on the sphere of the fixed stars by its apparent diameter (~30')
  - solar perturbations induces variations of the length of the lunation. Differences can amount to up to ±7h

Moon and tides

The rotation of the earth and the moon are synchronized

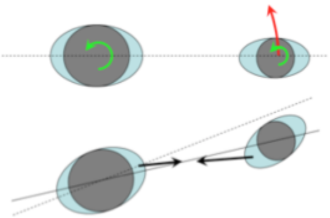
- the moon shows always the same face modulo some small oscillations
- amplitude of these oscillations 3.5' : librations
- rotation and revolution periods are equal to within 0.1s
- reason : the moon is misshaped due to tidal interactions. Flattened ellipsoid aligned towards the earth (stable equilibrium position)

+ physical and optical librations (w/ latitude due to the inclination of the axis of the moon/ perpendicular of the orbit, w/ longitude because of the law of equal area)

Spin-orbit coupling and tidal locking

Rotation of the satellite faster than the revolution around the planet

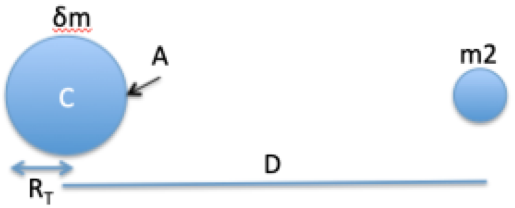
- Torques slow down the rotation of the planet
- Angular momentum conservation ⇒ increase of the distance satellite-planet



Tidal effect

Tidal force ⇒ difference between the force at the surface and at the center of the object (planet, moon, cluster of stars ...)

Geometry of the system :



Tidal effect

Gravitational force @ A :

$$F_A = G\frac{m_2 \delta m}{(D - R_T)^2} \sim G\frac{m_2 \delta m}{D^2}(1 + 2\frac{R_T}{D})$$

and at point C :

$$F_C = G\frac{m_2 \delta m}{D^2}.$$

Then tidal force is :

$$F_t = F_A - F_C = 2G\frac{m_2 \delta m}{D^3} R_T \propto \frac{1}{D^3} \cdot m_2 \cdot R_T.$$

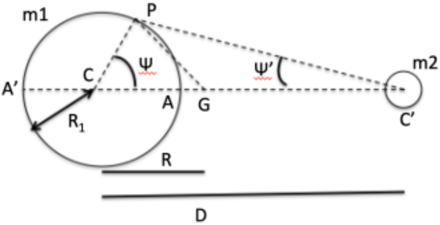
Tidal effect

In the case of the system Earth-Moon :

- In the case of the earth, F_t is only 2.2 times smaller for the Sun than for the moon !
- The plane of the moon-earth orbit is only inclined by 5° w/ the ecliptic, the two effects do combine.
- BUT for a proper analysis, we can not neglect the rotation around the centre of mass (otherwise free-fall) : one must take into account the centrifugal force.

Case with rotation

Geometry :



Tidal effect

We consider a rotating frame rotating around the point G.

We will assume that the orbital motion is circular (and Ω =constant) + rotation and revolution are synchronized.

We note :

$$\overrightarrow{\Omega} = \Omega \overrightarrow{e_z}$$

and

$$\overrightarrow{CC'} = D \overrightarrow{e_{CC'}}.$$

Tidal effect

Lets work out the forces of a mass element located @ P. It feels :

1. the gravitational force due to m_1 : unimportant for tidal effect
2. the gravitational force due to m_2 : $\overrightarrow{F_2} = G\frac{m_2}{\overline{PC'}^2}\overrightarrow{e_{PC'}}$

   where $\overline{PC'}^2 = D^2 - 2DR_1\cos\Psi + R_1^2$

   and $\overrightarrow{e_{PC'}} = \cos\Psi'\overrightarrow{e_{CC'}} - \sin\Psi'\overrightarrow{e_\Psi}$.

   Since $R_1 \ll D$, we have $\cos\Psi' \sim 1$ and $\sin\Psi' \sim \Psi' \sim \frac{R_1}{D}$.

   So to first order in R_1/D, F_2 writes

   $\overrightarrow{F_2} = G\frac{m_2}{D^2}(1+2\frac{R_1}{D}\cos\Psi)\overrightarrow{e_{CC'}} - G\frac{m_2}{D^2}\sin\Psi'\overrightarrow{e_\Psi}$

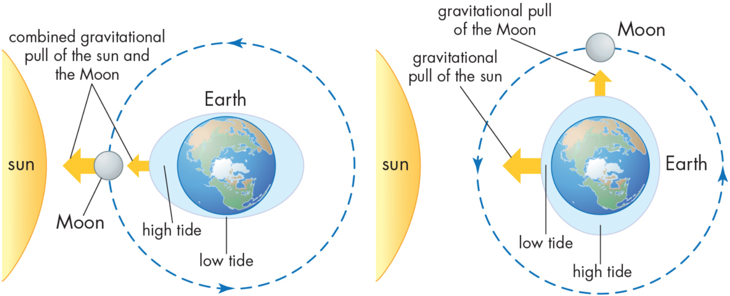
Tidal effect

3 the centrifugal force
$-\overrightarrow{\Omega} \wedge (\overrightarrow{\Omega} \wedge \overrightarrow{GP}) = -\overrightarrow{\Omega} \wedge (\overrightarrow{\Omega} \wedge \overrightarrow{GC}) - \overrightarrow{\Omega} \wedge (\overrightarrow{\Omega} \wedge \overrightarrow{CP})$
The first term $-\Omega^2 R\overrightarrow{e_{CC'}}$ compensates $G\frac{m_2}{D^2}\overrightarrow{e_{CC'}}$ of $\overrightarrow{F_2}$ because the system is stationary.
The second term is due to the rotation of object 1 onto itself : cause of flattening

(1)+(2)+(3) $\Rightarrow\propto \frac{m_2}{D^3}R_1$ and is oriented towards AA' (stretching).

Typically 2 tides in 24h on the Earth.

Tidal effect



Other technique for treating tides : the potential approach

A fluid surface adjust itself on the equipotential lines ⇒ compute the potential in a frame rotating at angular speed Ω.

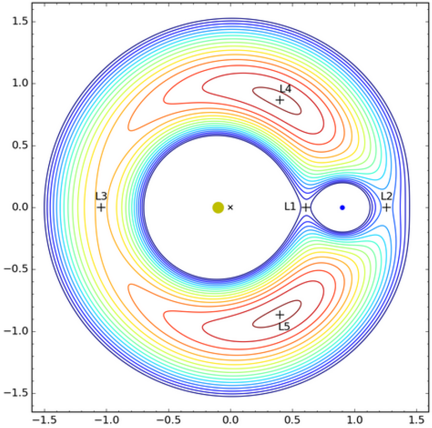
Moon-earth system : equipotential surfaces close to the earth are ellipsoids of revolution aligned on the earth-moon axis.

Height of the local tide modified by :

- inclination of the polar axis
- position of the Sun
- limits of the seas

Potential energy of a test particle of unit mass in the rotating frame : $\Psi = -G\frac{m_1}{r_1} - G\frac{m_2}{r_2} - \frac{1}{2}\Omega^2 r^2$

Potential approach



Potential approach

Points noted L# are Lagrangian points (extremas of the potential in the rotating frame).

- L1, L2 and L3 are unstable points (saddle points)
- L4 and L5 are stable

The equipotential passing through L1 defines the **Roche Lobes**.

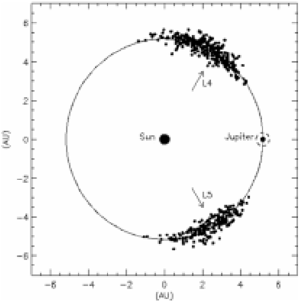
Potential approach

Application of the potential approach :

- satellite positioning (restricted three bodies problem because satellite does not influence the motion of the planets and stars + Coriolis force due to satellite velocity / frame)
- mass transfer in binaries
- tidal tails in galaxies collision
- ...

Potential approach

Here is an example : Trojan asteroids location for the Sun-Jupiter system



Other examples of the importance of tides : galaxy-satellite encounter (Toomre & Toomre 1972)

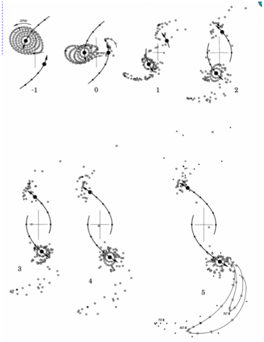
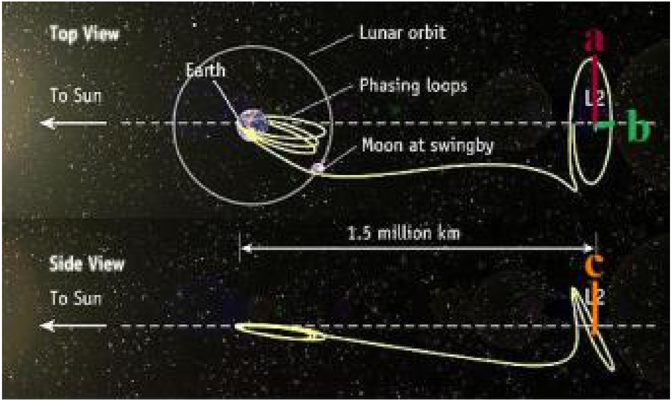


FIG. 2.—A flat direct ($i = 0°$) parabolic passage of a companion of equal mass

Potential approach

Launching a satellite :



Units and coordinate systems

CGS & SI

In astrophysics with use a combination of CGS and SI units as well as units intrinsic to astronomy.

CGS = centimeter / gram / second

- Dates back to 1860, used in astrophysics

SI = international system → uses meter and kilograms

- Adopted in 1961

Meter and second are unfortunately not adapted to most of our systems ! We use some units that are proper to astrophysics :

- Length unit : parsec (trigonometric distance to nearby objects), AU (astronomical unit)
- Time unit : year (yr), Myr, Gyr

IAU recommandations

Table 1. The names and symbols for the SI base and supplementary units.

Quantity	*SI Unit: Name*	*Symbol*
length	metre	m
mass	kilogram	kg
time [1]	second	
electric current	ampere	A
thermodynamic temperature	kelvin	K
amount of substance	mole	mol
luminous intensity	candela	cd
plane angle	radian	rad
solid angle	steradian	sr

[1] The abbreviation sec should not be used to denote a second of time.

Table 2. Special names and symbols for SI derived units.

Quantity	*SI Unit: Name*	*Symbol*	*Expression*
frequency	hertz	Hz	s^{-1}
force	newton	N	kg m s^{-2}
pressure, stress	pascal	Pa	N m^{-2}
energy	joule	J	N m
power	watt	W	J s^{-1}
electric charge	coulomb	C	A s
electric potential	volt	V	J C^{-1}
electric resistance	ohm	Omega	V A^{-1}
electric conductance	siemens		A V^{-1}
electric capacitance	farad	F	C V^{-1}
magnetic flux	weber	Wb	V s
magnetic flux density	tesla	T	Wb m^{-2}
inductance	henry	H	Wb A^{-1}
luminous flux	lumen	lm	cd sr
illuminance	lux	lx	lm m^{-2}

IAU recommandations

Table 3. Examples of SI derived units with compound names.

Quantity	*SI unit: Name*	*symbol*
density (mass)	kilogram per cubic metre	kg m^{-3}
current density	ampere per square metre	A m^{-2}
magnetic field strength	ampere per metre	A m^{-1}
electric field strength	volt per metre	V m^{-1}
dynamic viscosity	pascal second	Pa s
heat flux density	watt per square metre	W m^{-2}
heat capacity, entropy	joule per kelvin	J K^{-1}
energy density	joule per cubic metre	J m^{-3}
permittivity	farad per metre	F m^{-1}
permeability	henry per metre	H m^{-1}
radiant intensity	watt per steradian	W sr^{-1}
radiance	watt per square metre per steradian	W m^{-2} Sr^{-1}
luminance	candela per square metre	cd m^{-2}

IAU recommandations

Table 4. SI prefixes and symbols for multiples and submultiples.

Submultiple	*Prefix*	*Symbol*	*Multiple*	*Prefix*	*Symbol*
10^{-1}	deci	d	10	deca	da
10^{-2}	centi	c	10^{2}	hecto	h
10^{-3}	milli	m	10^{3}	kilo	k
10^{-6}	micro	mu	10^{6}	mega	M
10^{-9}	nano	n	10^{9}	giga	G
10^{-12}	pico	p	10^{12}	tera	T
10^{-15}	femto	f	10^{15}	peta	P
10^{-18}	atto	a	10^{18}	exa	E

Note: Decimal multiples and submultiples of the kilogram should be formed by attaching the appropriate SI prefix and symbol to gram and g, not to kilogram and kg.

IAU recommandations

Table 5. Non-SI units that are recognised for use in astronomy.

Quantity	*Unit: Name*	*Symbol*	*Value*
time [(1)]	minute	min or "	60 s
time	hour	h	3600 s = 60 min
time	day	d	86 400 s = 24 h
time	year (Julian)	a	31.5576 Ms = 365.25 d
angle [(2)]	second of arc	"	(pi/648 000) rad
angle	minute of arc	'	(pi/10 800) rad
angle	degree	o	(pi/180) rad
angle [(3)]	revolution(cycle)	c	2pi rad
length	astronomical unit	au	0.149 598 Tm
length	parsec	pc	30.857 Pm
mass	solar mass	Mo	1.9891 x 10^{30} kg
mass	atomic mass unit	u	1.660 540 x 10^{-27} kg
energy	electron volt	eV	0.160 2177 aJ
flux density	jansky [(4)]	Jy	10^{-26} W m^{-2} Hz^{-1}

[1] The alternative symbol is not formally recognised in the SI system.

[2] The symbol mas is often used for a milliarcsecond (0".001).

[3] The unit and symbols are not formally recognised in the SI system.

[4] The jansky is mainly used in radio astronomy.

[5] The degree Celsius (oC) is used in specifying temperature for meteorological purposes, but otherwise the kelvin (K) should be used.

Warning : second and arcsecond

5.14 Time and angle : The units for sexagesimal measures of time and angle are included in Table 5. The names of the units of angle may be prefixed by 'arc' whenever there could be confusion with the units of time. The symbols for these measures are to be typed or printed (where possible as superscripts) immediately following the numerical values; if the last sexagesimal value is divided decimally, the decimal point should be placed under, or after, the symbol for the unit; leading zeros should be inserted in sexagesimal numbers as indicated in the following examples.

2d 13h 07m 15.259s 06h 19m 05.18s 120o 58' 08".26

These non-SI units should not normally be used for expressing intervals of time or angle that are to be used in combination with other units.

Time reference frames

The second of time

SI definition of the second : bureau international des poids et mesures (1967)

A second is the duration of 9 192 631 770 periods of radiation of the transition between two hyperfine levels of the fundamental state of the cesium 133 atom.

Before, terrestrial motion served as reference

IAU Definition : 86400 s = 1 d
1 Julian year= 31 557 600s = 365.25d

Time

- Origine of time : January 1st 2000 , 12h UTC (J2000) = **standard epoch**
- By definition, the beginning of a Julian year is separated from the standard epoch by an integer number of Julian year. This leads to a progressive gap w/ civil calendar (Gregorian)
  - Jan. 1st 2000 @ 12h UTC : Beginning of Julian year 2000 + Julian day 2451545.0
  - Beginning of Julian year $2005 = 2451545.0 + 5 \times 365.25 = 2453371.25 =$ December 31 2004 @ 18h.
  - Before 1984 , Bessel year (standard epoch B1950.0) was used

Time

TAI : **International Atomic Time** network of ~200 clocks in ~50 laboratories complying with the atomic definition of the second.

Mesurements are combined in Sèvres at the bureau international des poids et mesures using a fixed algorithm (defined in 1971)

ET : **Ephemeris Time** based on Newton's law and the inertia principle (time is absolute and uniform).

Theory based on the motion of the earth around the Sun using a force model as complete as possible. A relation (IAU 1952) defines ET at the Sun's longitude which is measurable.

TT : relativistic time introduced in 1991, comes from the application of GR to the solar system.

ET is replaced by this **Terrestrial Time (TT)**, close from the proper time of the earth geoid (equipotential surface). Distinct from the geocentric time-coordinate which is the proper time of a clock at the centre of the earth, free from the earth potential.

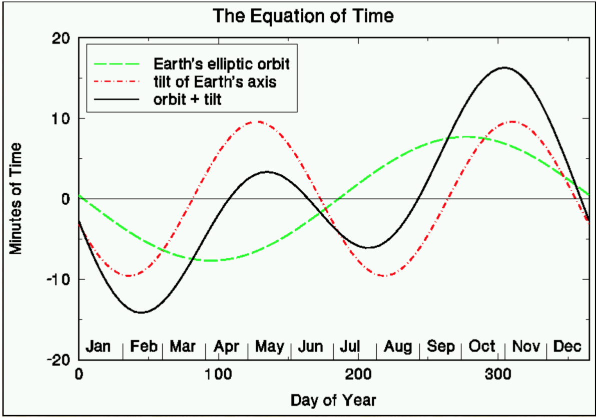
Solar Time and Universal Time (UT) :

at a given time and place, the hour angle of the Sun $T_\odot$ is the true Solar time one writes :

$$T_\odot = A + B.t - E + \tau$$

- $A + B.t$ is the uniform part of $T_\odot$ (A and B are constants, e.g. TAI)
- E is the equation of time : sum of the predictable non-linear part of the true Solar time (mostly due to the law of equal area)
- τ comes from the inequalities of the earth's rotation around its axis that are hard to model (secular slow down, fluctuations...)

The equation of time :



Mean Solar Time :

$$T_m = T_\odot + E = A + B.t + \tau$$

On Greenwich meridian, T_m gives universal time (UT) as UT=Tm+12h (to have UT=0h @ midnight)

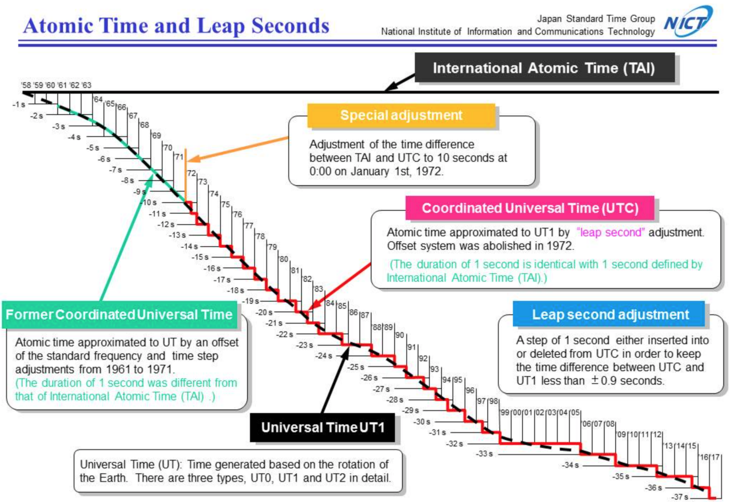
UT is uniform only if one neglects τ !

IERS publishes UT1. There, the pole entering the definition of the angles is the true celestial pole of the earth that moves on its surface in an unpredictable manner.

Coordinated Universal Time (UTC) :

- TAI being separate from the terrestrial motion, the mean day does not contain exactly 24x3600s of TAI...
- irregularities of the rotation of the earth prevent ajusting the duration of a second (does not hold over long time scales)
- an hybrid time was created that permits to keep the link between the atomic time and the earth orientation using adequate slips : TAI-UTC=n seconds (n integer) $\rightarrow$ preserves the uniformity of the atomic time by blocks

Time


Atomic Time and Leap Seconds
Japan Standard Time Group
National Institute of Information and Communications Technology
NICT
International Atomic Time (TAI)
Special adjustment
Adjustment of the time difference between TAI and UTC to 10 seconds at 0:00 on January 1st, 1972.
Coordinated Universal Time (UTC)
Atomic time approximated to UT1 by "leap second" adjustment. Offset system was abolished in 1972.
(The duration of 1 second is identical with 1 second defined by International Atomic Time (TAI).)
Former Coordinated Universal Time
Atomic time approximated to UT by an offset of the standard frequency and time step adjustments from 1961 to 1971.
(The duration of 1 second was different from that of International Atomic Time (TAI) .)
Leap second adjustment
A step of 1 second either inserted into or deleted from UTC in order to keep the time difference between UTC and UT1 less than ±0.9 seconds.
Universal Time UT1
Universal Time (UT): Time generated based on the rotation of the Earth. There are three types, UT0, UT1 and UT2 in detail.

Time

Civil time : for a given location, it is the mean solar time T_m+12h

- UT is the civil time of Greenwich

Legal time : time used in a given country : UTC +n hours with n integer.

Case of France :

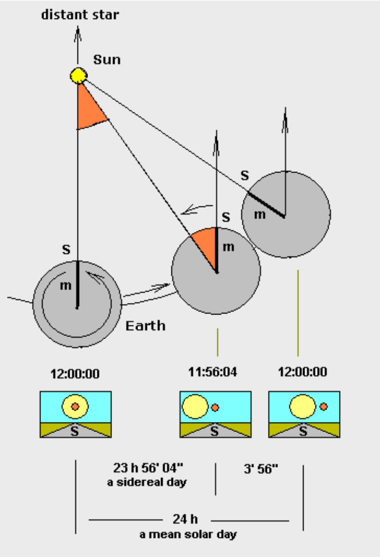
- UTC + 1h in winter
- UTC + 2h in summer

Sidereal day : time interval between two transits of the vernal point at the local meridian = 23h 56m 4.090s UT

Stellar day : mean time interval between two transits of a star at the meridian

⚠ These two definitions differ by 0.0083s because the vernal point has a retrograde motion on the ecliptic 50.26"/365.25d

Sidereal day :



Time

Sidereal time : hour angle of the vernal point. This time is local and non uniform

Published by bureau des longitudes for Greenwich @ 0h UT every day

- 1s of $T_m = 1.002379s$ of sidereal time
- as each day contains $24 \times 3600s$, it is also the ratio between the duration of a mean solar day to the mean sidereal time
- $1 \text{ day}_{sid} = 1 \text{ day}_{\odot .mean} - 3m55.9s$ of mean solar time

Years and Calendars

- 1 Julian year=365.25 days of 24x3600s of TAI
- Sidereal year : time taken by the Sun to orbit the Sun once (360° on the ecliptic)
- 1 sidereal year = 365.25636 days of UT (or true solar time)
- Tropical year : time interval between two march equinoxes (conjunction between the Sun and the vernal point)

  1 tropical year= 365.242189 days of UT$<$ 1 sidereal year because of the retrograde motion of the vernal point 50"/yr

Years and Calendars

- Anomalistic year : time interval between two passages of Earth at perihelion ($>$ sidereal year because the perihelion moves slowly in the prograde direction)
- Civil year : 365 days of UT for normal year, 366 for leap years (induces a mean year of 365.25 days$>$ tropical year).

  To compensate the gaps, year that are multiple of 100 are NOT leap year unless they are multiple of 400...

Coordinate systems

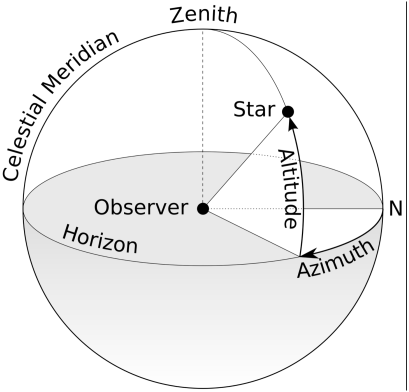
Sphere of fixed stars

- Viewed from earth the relative positions of distant objects are constant during the year (distant meaning $d \gg 1$ AU). They define the **sphere of fixed stars**.
- The apparent motion of nearby objects projects onto this sphere.

Azimuthal coordinates (horizontal system)

- Azimuth : A
- Altitude/elevation : h

- Coordinates depend on the location !
- Coordinates depend on time !



Azimuthal coordinates (horizontal system)

- A and h : angular coordinates
- A varies from 0 to 360° from the south meridian towards the west
- h varies from 0 to 90°
- Zenital distance : z=90° - h

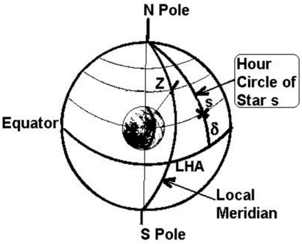
Useful for observations :

- Obstacles to observation/pointing
- Computation of airmass
  - 1 at zenith
  - $1/\cos(z)$ at the object position

Hour angle coordinates

Frame defined by the equatorial plane and the meridian passing through the poles and the zenith

- Hour angle LHA (or H in hours) increasing towards the west
- Declination δ from -90° to 90°



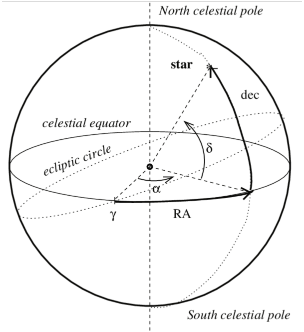
Hour angle coordinates

- LHA depends on the location BUT δ does not depend on the location nor on the time
- true only if we neglect the polar axis variations (precession & nutation)
- star at the highest when LHA=0
- Local sidereal time is the hour angle of the vernal point

Equatorial coordinates

Frame defined by the equatorial plane and the earth axis. Origin at the vernal point (direction of the Sun at march equinox)

- Right ascension : α in hours minutes seconds increasing towards East Declination : δ



Equatorial coordinates

- Coordinates independent of location
- Location of the vernal point varies slowly with time because of precession. A correction is needed w/ observation date.
- Sidereal time (indicated in control room) T=H+α for all celestial bodies
- For bodies close to zenith :
  - $\delta \sim$ latitude of the telescope
  - $\delta \sim$ local sidereal time (H=0)

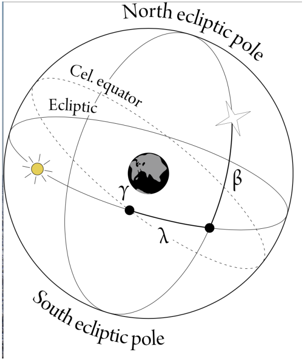
Ecliptic coordinates

Reference plane= ecliptic plane

- ecliptic longitude λ from 0 to 360°
- ecliptic latitude β from -90 to 90°

Reference frame for solar system objects and zodiacal light studies

zodiac $= \pm 8.5°$ strip around the ecliptic plane. Contains the apparent trajectories of solar system planets

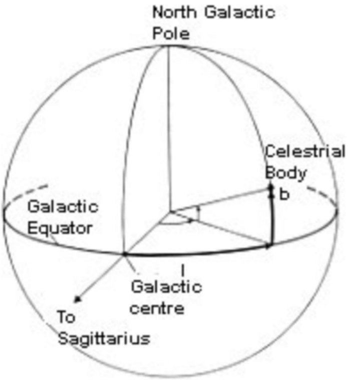


Galactic coordinates

Reference plane= Galactic plane

- longitude ℓ from 0 to 360°
- latitude b from -90 to 90°

- Origin : direction to the Galactic center
- ℓ=+90° towards Galactic rotation



This is the reference frame for galactic studies

Equatorial coordinates J2000.0 for Galactic reference points :
From COSMOS @ Swinburne University

	Right Ascension	Declination
North Galactic pole +90° latitude	12h 51m 26.00s	+27° 7' 42.0"
South Galactic pole -90° latitude	0h 51m 26.00s	-27° 7' 42.0"
Galactic centre 0° longitude, 0° latitude	17h 45m 40.04s	-29° 00' 28.118"
Galactic anti-centre 180° longitude, 0° latitude	6h 17m 0.0s	+22° 30' 0.0"

Supergalactic coordinates

Reference plane= approximative plane structure containing the local galaxies cluster, Virgo, Centaurus and Hydra clusters (~1950)

- 0 point : intersept of this plane with the Galactic plane
- longitude : SGL
- latitude : SGB
- Origin of longitudes : ℓ=137.7° (b=0°) or α=2h 49m 14s,δ=51° 39' 42"
- North supergalactic pole : α=18h 55m 01s, δ= 15° 42' 32"

Summary

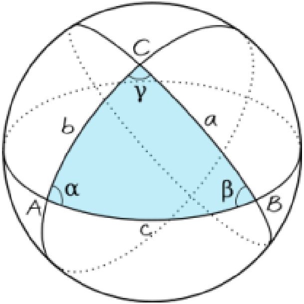
Coordinate system	Center	Fundamental plane (0° latitude)	Poles	Longitude	Latitude	Primary direction (0° longitude)
Horizontal (Azimuthal)	Observer	Horizon	Zenith, Nadir	Azimuth A	Altitude-elevation h	Intersection of local meridian and horizon
Hour-angle	Earth	Celestial equator	Celestial poles	Local hour angle LHA or H	Declination δ	Intersection of local meridian and equator
Equatorial	Earth	Celestial equator	Celestial poles	Right ascension α	Declination δ	Vernal equinox
Ecliptic	Earth	Ecliptic plane	Ecliptic poles	Ecliptic longitude λ	Ecliptic latitude β	Vernal equinox
Galatic	Sun	Galactic plane	Galactic poles	Galactic longitude ℓ	Galactic latitude b	Galactic center (SgrA*)
Super-galactic	Sun	Super-galactic plane	Super-galactic poles	super-galactic longitude SGL	Supergalactic latitude SGB	Intersection of super-galactic plane and galactic plane

Spherical trigonometry basics

Spherical trigonometry

To change reference frame, we use the relations of spherical trigonometry. One define

- α, β and γ are the angles at the vertices
- a, b and c are the sides (angles between the vertices and the centre O)



Basic relations

- cosine rules (circular permutation)

  $$\cos c = \cos a \cos b + \sin a \sin b \cos \gamma$$

- supplemental relation

  $$\cos \gamma = -\cos \alpha \cos \beta + \sin \alpha \sin \beta \cos c$$

- sine rules

  $$\frac{\sin a}{\sin \alpha} = \frac{\sin b}{\sin \beta} = \frac{\sin c}{\sin \gamma}$$

Basic relations

- Gauss group

$$\begin{cases} \sin a \cos \beta = \cos b \sin c - \sin b \cos c \cos \alpha \\ \sin a \sin \beta = \sin b \sin \alpha \\ \cos a = \cos b \cos c + \sin b \sin c \cos \alpha \end{cases} \quad (1)$$

Note that these formula can be derived from the scalar product.

On wikipedia you will find additional material but Gauss group + the cosine rules are generally sufficient to solve our problems.

Example

Lets note :

- λ_0, observer's longitude on the earth
- φ_0, observer's latitude on the earth
- ϵ, obliquity of the ecliptic (about 23.4°)
- θ_L, local sidereal time
- θ_G, Greenwich sidereal time
- (α_G, δ_G) the equatorial coordinates of the north Galactic pole (192.85948°, 27.12885°, J2000)
- ℓ_{NCP}, the Galactic longitude of the north celestial pole (122.93192°)
- $H = \theta_L - \alpha$ the hour angle

Example

Hour angle ↔ right ascension

$$h = \theta_L - \alpha \quad \text{or} \quad h = \theta_G + \lambda_0 - \alpha$$
$$\alpha = \theta_L - h \quad \text{or} \quad \alpha = \theta_G + \lambda_0 - h$$

Equatorial ↔ Galactic

$$\begin{cases} \cos(l_{NCP} - l)\cos(b) = \sin(\delta)\cos(\delta_G) - \cos(\delta)\sin(\delta_G)\cos(\alpha - \alpha_G) \\ \sin(l_{NCP} - l)\cos(b) = \cos(\delta)\sin(\alpha - \alpha_G) \\ \sin(b) = \sin(\delta)\sin(\delta_G) + \cos(\delta)\cos(\delta_G)\cos(\alpha - \alpha_G) \end{cases}$$

and the inverse transfrom

$$\begin{cases} \sin(\alpha-\alpha_G)\cos(\delta) = \cos(b)\sin(l_{NCP}-l) \\ \cos(\alpha-\alpha_G)\cos(\delta) = \sin(b)\cos(\delta_G) - \cos(b)\sin(\delta_G)\cos(l_{NCP}-l) \\ \sin(\delta) = \sin(b)\sin(\delta_G) + \cos(b)\cos(\delta_G)\cos(l_{NCP}-l) \end{cases}$$

Equatorial ↔ Azimutal

$$\begin{cases} \sin h = \sin\varphi_0 \sin\delta + \cos\varphi_0 \cos\delta \cos H \\ \cos h \cos A = -\cos\varphi_0 \sin\delta + \sin\varphi_0 \cos\delta \cos H \\ \cos h \sin A = \cos\delta \sin H \end{cases}$$

$h > 0$ is the observability condition

Structure and history of the Universe

Important steps

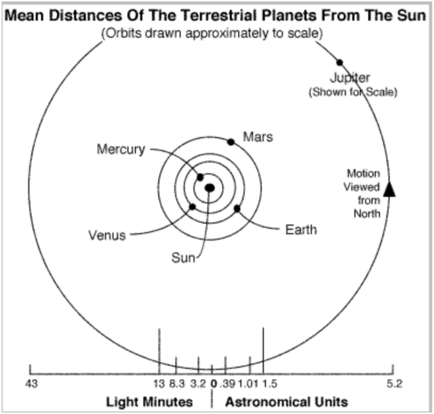
- **January 1st :** Big Bang (-13.7 Gyr)
- **May 1st :** Birth of the Milky Way (-10 Gyr)
- **September 9th :** Birth of the solar system (-4.7 Gyr)
- **September 14th :** Birth of the Earth (-4.5 Gyr)
- **October 2nd :** origins of life (Cyanobacteries, -3.7 Gyr)
- **November 25th** - **December 17th :** oxygenation of the atmosphere (-2.7 … -0.6 Gyr)
- **December 26th :** first dinosaurs (-230 Myr)
- **December 31st :**
  - 13h30 : ancestries of monkeys (-20 Myr)
  - 22h30 : first homo habilis (-2.7 Myr)
  - 23h59m56s : year 0 (-2000 yr)
  - 23h59m59s : start of space exploration

Link to the lifetime of astonomical clocks

- Big Bang (-13.7 Gyr) : Age of the universe. Lifetime of a $\sim 0.9 M_{\odot}$ star.
- Birth of the Milky Way (-10 Gyr) : Lifetime of a solar type star.
- Origin of the solar system (-4.7 Gyr)
- Earth formation (-4.5 Gyr)
- Appearance of life (-3.7 Gyr)
- Oxygenation of the atmosphere (-2 Gyr) : Lifetime of a $\sim 2 M_{\odot}$ star
- First dinosaurs (-250 Myr) : 1 Galactic revolution
- Ancestries of Monkeys (-20 Myr)
- First homo habilis (-2.7 Myr) : lifetime of a $100 M_{\odot}$ star.

Solar system

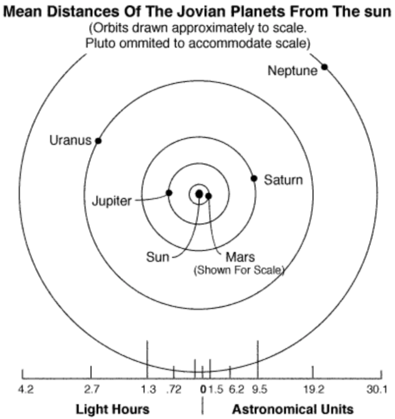
Terrestrial and Jovian planet orbits



NASA

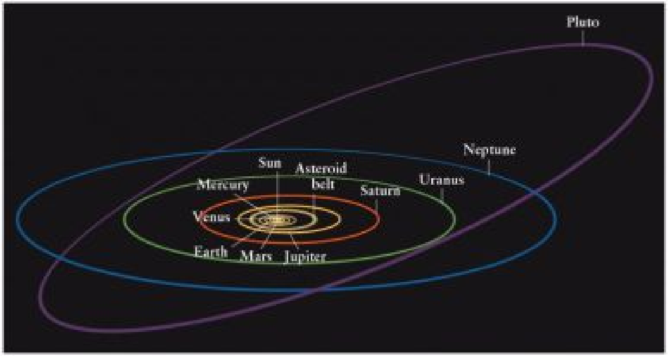
- Mean radius of the Earth orbit : 1 AU ~ 150 Mkm
- Terrestrial (rocky) planets up to mars.

Terrestrial and Jovian planet orbits



- Jovian planets (gaseous, giant) from Jupiter to Neptune

Solar system



- Size of the solar system (diameter) : 100 AU (10^{13} km).

Useful order of magnitudes :

Useful order of magnitudes :

- the Sun's radius is roughly 100 times larger than that of the earth
- the moon's orbit ($R \sim 4.10^5$ km) fits in the Sun
- 1 AU is 100 times the Sun's diameter.

Titus-Bode law :

Predicts the size of the orbit of planets (1766 and not really a law)

$$r = 0.4 + 0.3 \times 2^{n-1}$$

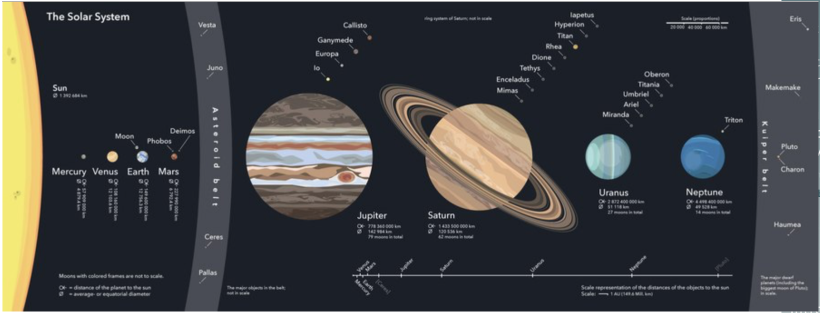
with r the radius of the orbit in AU and n the rank of the planet ($-\infty$ for Mercury, 1 for Venus ...).

Titus-Bode law

Titus-Bode law and planet orbits parameters in AU (source Wikipedia)

Planet	Rank	Predicted distance	Semi-major axis	Perihelion	Aphelion	Eccentricity	Absolute error	Relative error
Mercury	$-\infty$	0.4	0.387	0.307	0.467	0.206	0.013	3.4%
Venus	1	0.7	0.723	0.718	0.728	0.007	0.023	3.2%
Earth	2	1.0	1.000	0.983	1.017	0.017	0.000	0.0%
Mars	3	1.6	1.523	1.381	1.665	0.093	0.077	5.1%
Ceres	4	2.8	2.765	2.547	2.983	0.079	0.035	1.3%
Jupiter	5	5.2	5.203	4.953	5.453	0.048	0.003	0.1%
Saturn	6	10.0	9.537	9.022	10.052	0.054	0.463	4.9%
Uranus	7	19.6	19.229	18.325	20.133	0.047	0.371	1.9%
Neptune	8	38.8	30.069	29.798	30.340	0.009	8.731	29.0%

Sizes and scales



©Wikipedia

Definition of a planet

IAU resolution August 2006 :

(1) A classical planet is a celestial body that a) is in orbit around the Sun, b) has sufficient mass for its self-gravity to overcome rigid body forces so that it assumes a nearly round shape, and c) has cleared the neighborhood around its orbit.

(2) A dwarf planet is a celestial body that verifies a), b) but not c).

(3) All other objects except satellites orbiting planets shall be referred to collectively as Small Solar-System Bodies.

(4) Pluto is a dwarf planet by the above definition [...].

Moons and small planets

The size and mass does NOT distinguish between planet and satellite (moon)

Moons

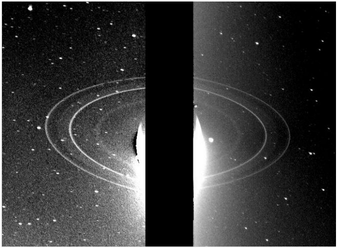
Jovian (giant) planets have numerous satellites !

Planet	Mercury	Venus	Earth	Mars	Jupiter	Saturn	Uranus	Neptune
# Moons	0	0	1	2	79	62	27	14

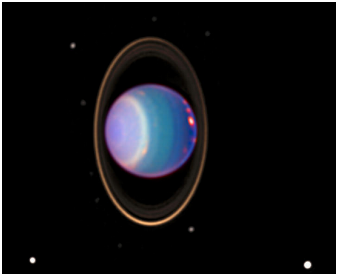
Rings

The four giant planets do have rings !

Neptun



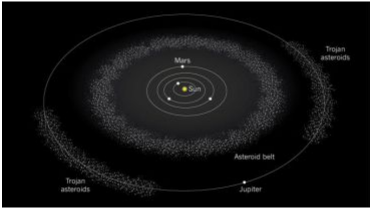
Uranus



Asteroids and comets

Asteroids are small solar system objects (SSS) according to IAU

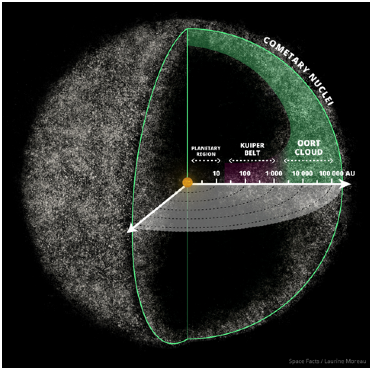
- Main belt between Mars and Jupiter ($R \sim 2.7$ AU)
- Trojan asteroids at the Lagrange points of Jupiter's orbit ($R \sim 5.2$ AU).
- Asteroids are mostly made of metal and rocky material.



Asteroids and comets

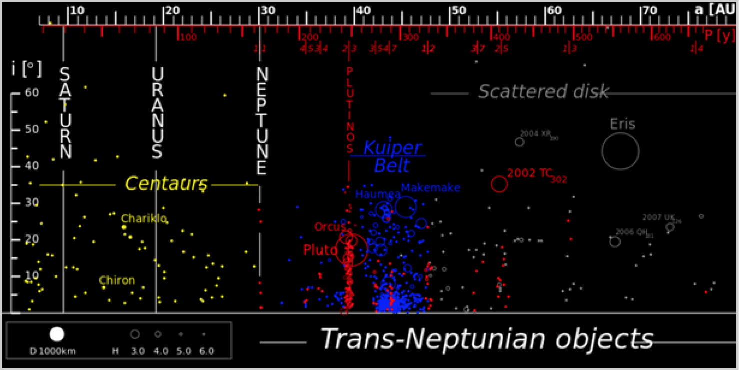
Comets are further away and originate in the Kuiper belt or even further out in Oort's cloud

- Comets are made essentially of icy material.



Asteroids and comets

Trans-Neptunian objects (TNO's) :



Mostly icy objects with organic compounds. Some are associated to resonances with planet orbits.

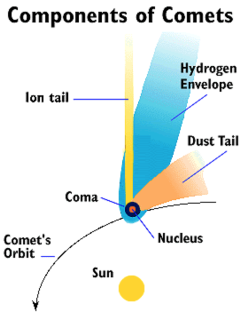
Comets and tails

A comet has more than just one tail ! Example Hale-Bopp

Comets and tails

A comet's tails consists of three parts :

- ion tail : the lightest material, tail oriented towards the Sun
- Hydrogen envelope
- Dust tail : heavy material, low kinetic energy



A small body (a fragment of a comet's tail) becomes a meteorite once entering the atmosphere.

Formation of the Solar system

- From the formation of the Sun to the formation of planets, differentiation included : about ~200 millions years.
- Planetary differentiation is the process of separating out different constituents of a planetary body. This process results in the planetary body being stratified with different layers of different density.
- After this period, a long spatial reorganizing phase : planetary migrations due to interactions with other planets and the debris disks ; stabilization (partial) in resonant locations ; "late heavy bombardment" (moon craters date back ≈700 Myr after the formation of the earth and moon).

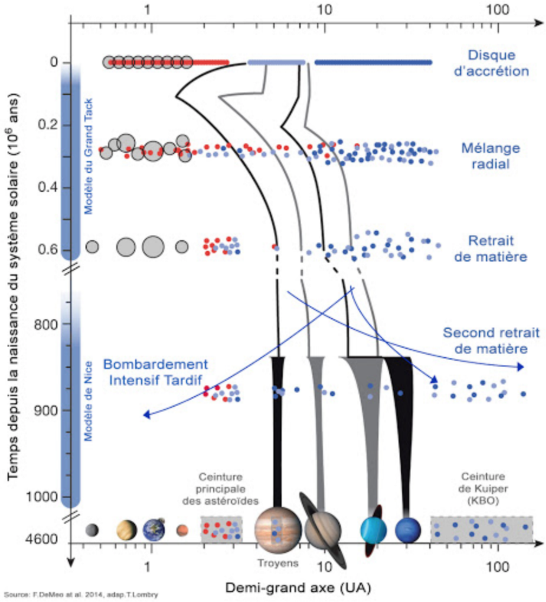
Formation of the Solar system

The most fundamental notion entering the formation of the solar system is that of orbital resonances. The most advanced model to date is the Nice model (Gomes et al. 2005, Nature) : can explain the Lunar seas, Oort cloud and Trojans on Jupyter's orbit.

The process of the solar system formation started ~4.6 Gyr ago :

- ~50 Myr : collapse of the gas cloud and formation of the Sun
- between 1 and 10 Myr : formation of planetary embryos
- up to 10 Myr : formation of giant planets
- 60 Myr : formation of the earth-moon system
- 100 Myr : formation of terrestrial planets
- 150 Myr : planetary migration

Formation of the Solar system



Zodiacal light

The zodiacal light is solar light diffused by dust in the ecliptic plane. Remanent of solar system formation.

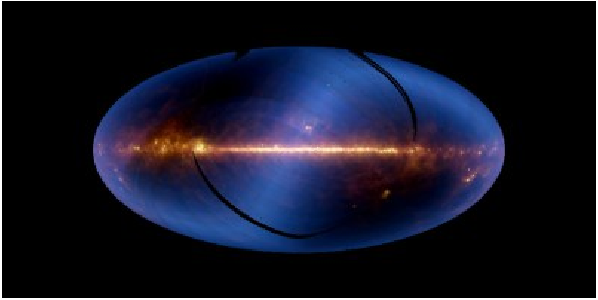


Image : IRAS (Infrared satellite, 1983, USA,UK,NL).
Blue : 12 μm, Green : 60 μm, Red : 100 μm.
Zodiacal light appears mostly at 12 microns, while thermal emission of dust grains, heated by local radiation is dominant above

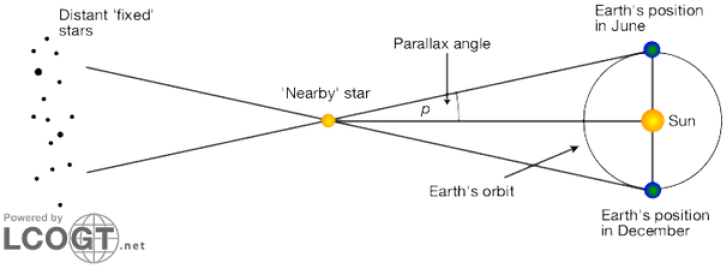
Measuring distances

Measuring distances : parallaxes

One of the main difficulties in astronomy is that the objects of interest are projected onto the 2D sky. Measuring distances is not an easy task.

For nearby objects, there exist a trigonometric method that allows to measure accurately distances based on the revolution of the earth around the Sun : the trigonometric parallax.

Measuring distances : parallaxes



Measuring distances : parallaxes

The parallax defines the unit parsec (pc) which is a distance unit : a star is at 1pc if its parallax is 1".

$$\begin{aligned} 1" &= 1 \text{ arcsecond} \\ &= 1/60e \text{ arcmin} \\ &= 1/3600 \text{ degree} \\ &= \frac{(\pi/180)}{3600} \text{ radian} \end{aligned}$$

π being always small one can use $\tan(p) \sim p = 1/D$ where D is the distance.

Measuring distances : parallaxes

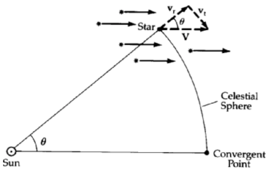
- The parallax is an apparent motion that adds up to the real motion of the star (the proper motion)
- The earth's atmosphere creates an aberration that must be corrected for.
- It is the only measurement that does not involve a physical model.

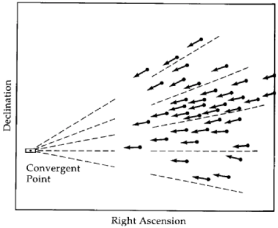
Measuring distances : parallaxes

Parsec compared to other distance units :

- $1\text{pc} = 1\text{AU}/1\text{"}(\text{radians}) = 3.10^{16}\text{m}$
- $1\text{pc} \sim 3\text{ly}$

When stars are in a stable cluster system (eg its physical size is not changing), we can use the apparent motion of the stars within the cluster to estimate the distance





Measuring distances : moving cluster method

The velocity vector V is split in its two orthogonal components : the line of sight velocity v_r and the tangential velocity v_t.
The tangential velocity is related to the proper motion (observed) via

$$v_t = d \sin \mu \equiv d\mu.$$

d is the distance and μ the proper motion (the rate of change of the position on the sky).

In this last equation, if we know the proper motion and the tangential velocity, we can solve for the distance.

Measuring distances : moving cluster method

From geometric principles, we have $V = \frac{v_r}{\cos\theta}$ and hence

$$v_t = V\sin\theta = v_r\tan\theta.$$

Equating the two definitions of v_t we get

$$d = \frac{v_r\tan\theta}{4.74\mu},$$

where d is the distance in pc, v_r the LOS velocity in km/s, θ the angle to the convergence point and μ the proper motion in arcsec/yr.

The factor 4.74 is the convertion from 1 AU/yr = 4.74 km/sec.

Measuring distances : main sequence fitting

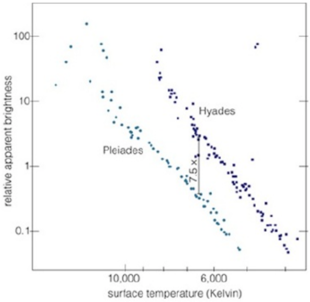
When distances to nearby stars were found using trigonometric parallaxes in the late 19th and early 20th century, it became possible to study the luminosities of stars.

Einar Hertzsprung and Henry Norris Russell both plotted stars on a chart of luminosity and temperature. Most stars fall on a single track, known as the Main Sequence, in this diagram, which is now known as the H-R diagram after Hertzsprung and Russell.

Nowadays the absolute magnitude is used instead of the luminosity, and the color is used instead of the temperature (see chapter on stellar evolution).

Measuring distances : main sequence fitting

When looking at a cluster of stars, the apparent magnitudes and colors of the stars form a track that is parallel to the Main Sequence, and by correctly choosing the distance, the apparent magnitudes convert to absolute magnitudes that fall on the standard Main Sequence.



Measuring distances : spectroscopic parallaxes

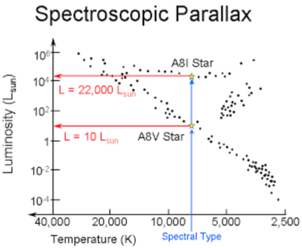
When the spectrum of a star is observed carefully, it is possible to determine two parameters of the star as well as the chemical abundances in the star's atmosphere.

The first of these two parameters is the surface temperature of the star, which determines the spectral type in the range OBAFGKM from hottest to coolest.

The second parameter that can be determined is the surface gravity of the star. The higher the surface gravity, the higher the pressure in the atmosphere. Stars with high surface gravity are called dwarfs while stars with medium gravity are called giants and stars with low gravity are called supergiants.

Measuring distances : spectroscopic parallaxes

Metallicity, gravity and temperature sets the absolute magnitude of the star (assuming it is a normal star). If one knows the apparent magnitude, we can then recover the distance to the star.



WARNING : radiation absorption by interstellar matter dims the flux of astronomical objects received on earth. This can cause systematic biases if extinction is not properly taken into account.

Measuring distances : Cepheids

At the beginning of the 20th century, Henrietta Leavitt discovered and studied a new type of variable star named Cepheids (from the prototype star delta Cepheid).

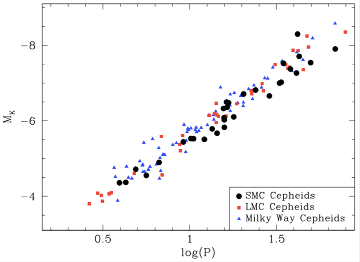
The brightness of these stars varies in a periodical manner and the period P scales with the brightness (the brighter the star, the longer the period).

H. Leavitt showed that a linear relation exists between the period and the magnitude of the Cepheid stars

$$< M >= a \log(P) + b$$

where $< M >$ is the absolute magnitude of the star and a and b are constants.

Measuring distances : Cepheids



The K-band Period-Luminnosity relation for the SMC stars (black) overplotted on the Milky Way (blue triangles) and LMC (red squares) samples.

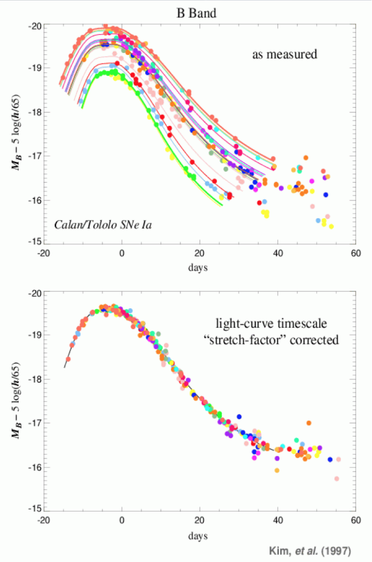
The two constants were measured using Cepheids whose distance was obtained using a different technique (such as the trigonometric parallax). Cepheids are useful standard candels as they are bright and can be observed even in distant galaxy clusters such as Virgo.

Measuring distances : type Ia supernovae

Type Ia supernovae are the explosions of white dwarf stars in binary systems, they are also called thermonuclear supernovae. They are not to be confused with core-collapse supernovae that result from the contraction of massive stars.

Accretion from a companion raises the mass above the maximum mass for stable white dwarfs, the Chandrasekhar limit. The white dwarf then starts to collapse, but the compression ignites explosive carbon burning leading to the total disruption of the star. The light output comes primarily from energy produced by the decay of radioactive nickel and cobalt produced in the explosion.

Measuring distances : type Ia supernovae



The peak luminosity is correlated with the rate of decay in the light curve : less luminous supernovae decay quickly while more luminous supernovae decay slowly.

When this correction is applied, the relative luminosity of a Type Ia SN can be determined to within 20% (~0.13 mag peak dispersion).

Measuring distances : type Ia supernovae

A few SNe Ia have been in galaxies close enough to us to allow the Hubble Space Telescope to determine absolute distances and luminosities using Cepheid variables, leading to one of the best determinations of the Hubble constant.

Type Ia supernovae can be seen to such great distances that one can measure the acceleration or curvature of the Universe using observations of faint supernovae ⇒ Nobel prize 2011 Permultter, Riess & Schmidt.

This list is far to be exhaustive !

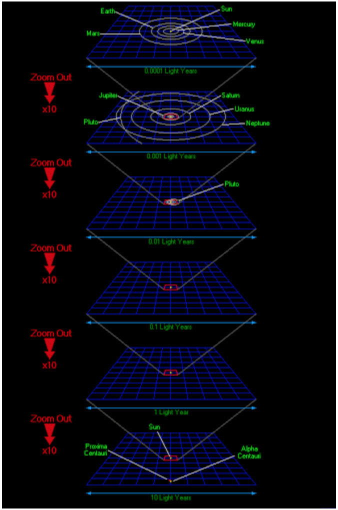
Other distance measurements : E.L. Wright website

Scales of the Universe

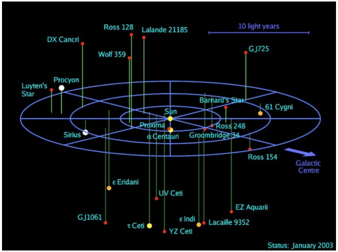
Local volume

The solar vicinity is almost empty !

There is very little material in the space surrounding the Sun, even planets do not fill the volume.



- The closest star is 4.2 ly away.
- Stars only fill a tiny volume of their parent galaxy.
- **Star-star collisions have a probability almost null, but in extremely dense regions (core of globular clusters).**



The Milky Way

Mapping the Milky Way is a hard task. Accurate distances can only be obtained via the measurement of parallaxes which is restricted to nearby stars.

Until recently, only a small fraction of the stars close to the Sun had a measurement of their parallax, with a precision of ~1 mas (milliarcsec), thanks to the Hipparcos satellite of ESA (1997). With Hipparcos, around 100,000 stars within 100 pc of the Sun had measurements down to an apparent magnitude of V~10.

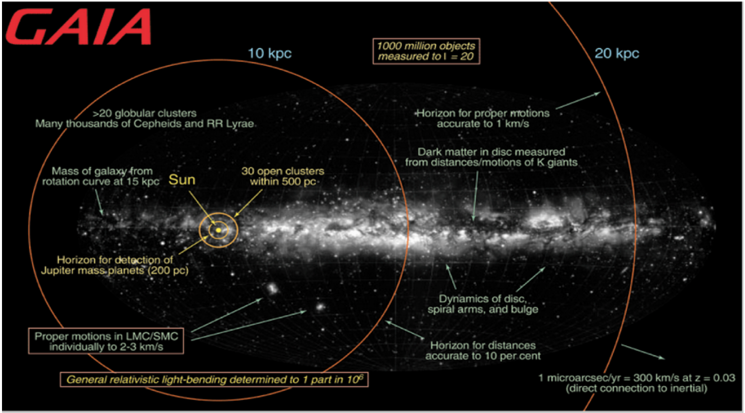
The Milky Way

Our knowledge of the Milky Way has recently improved with the release of the first two catalogs of the Gaia mission.

Gaia's goal is to measure the parallaxes, the photometry and the proper motions of stars (+ some additional parameters such as temperature, metallicity, extinction...) down to a magnitude G$\sim$20.

$\Rightarrow$ covers a large fraction of the Milky Way volume and bright stars in nearby galaxies.

The Milky Way



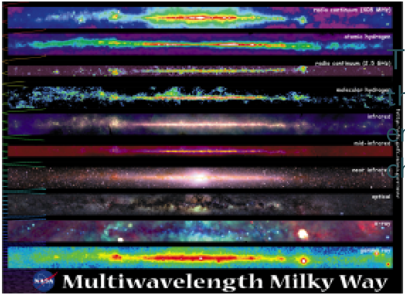
Currently we are at the third data release for this mission with astrometric information for about $1.46\,10^9$ stars down to G=21.

View of the Milky Way as seen from earth

One can easily see the black stripes in the Galactic disc which are due to extinction by interstellar material (dust). This material dims the light received from the stars and prevents easy measurement of distances because it changes the observed properties of an object (reddening).

The Milky Way

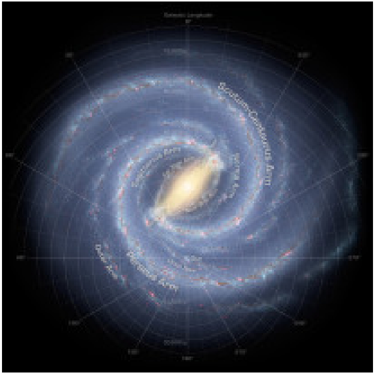
The appearance of the Galactic disc changes according to the wavelength we use for observations :



This is due to the fact that the physical process/source emitting the photons changes with wavelength.

The Milky Way

Top view of what could be the Milky Way (artist view)



We know it is a spiral galaxy and that there is a barred structure in its center (a galactic bar).

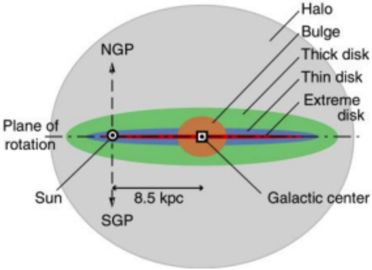
Its size is believed to be about 16 kpc in radius and its type in the Hubble classification is SBc which means a barred spiral galaxy (SB) with loosely wound spiral arms.

A more schematic view is given by this figure which shows the location of the main spiral structures as defined by the gas and the central bar.

The Milky Way

If one looks at the Galaxy from the side, its disc is extremely thin.

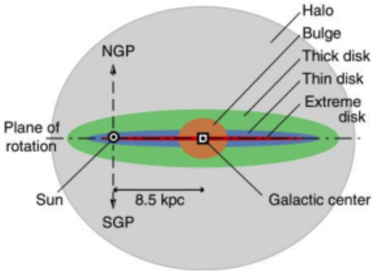
However, we can distinguish more than just one population.



The Milky Way

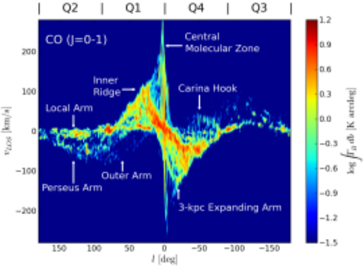
The disc can be decomposed of three main populations :

- The Thin disc (most of the mass)
- The thick disc which is an extended structure whose origin is yet unclear
- The extreme disc which is the site of star formation and is linked to the gas (very young stars in this disc).



The Milky Way

Quite a lot of information on the structure of our Galaxy is gained from the study of the gas (HI and CO) through analysis of so-called lv-plots :

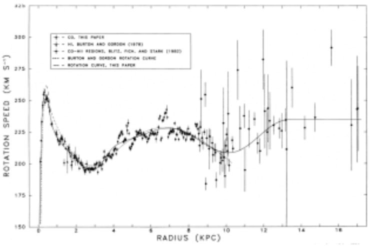


Structures in this diagram correspond to non axisymmetric features in the Milky Way.
One can see the signature of spiral arms, the central bar or the nuclear ring (CMZ).

Assuming the gas is on circular orbit (it is a collisional population) and that departure from circularity is negligible, one can construct the rotation curve of the Galaxy (assuming we know the distance to the Galactic center and the local rotation velocity) via the use of the tangent point method.

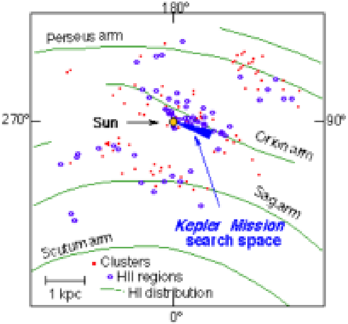
The Milky Way

One can note that beyond the position of the Sun, uncertainties are increased. This is due to the fact that the tangent point method does work only for point inside the solar circle, beyond tangent points do not exist anymore.



Some more information on the solar neighborhood :

The Sun is located close to spiral structures detected on the gas (HI and CO). The location of the spiral arms as defined by the gas matches the location of young objects such as open clusters or HII regions.



The Milky Way

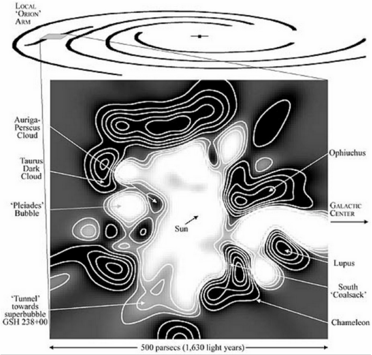
In terms of mass or column density, the solar neighborhood contains

Inventory of the Solar Neighbourhood

	Component	volume density	vertical velocity dispersion	surface density
		$M_\odot\ pc^{-3}$	$km\ s^{-1}$	$M_\odot\ pc^{-2}$
Gas	Molecular Hydrogen H_2	0.021	4.0	3.0
	Ionised Hydrogen H I	0.028	8.0	8.0
	warm gas	0.001	40.0	2.0
Stars	giants	0.0006	17.0	0.4
	$M_V < 2.5$	0.0031	7.5	0.9
	$2.5 < M_V < 3.0$	0.0015	10.5	0.6
	$3.0 < M_V < 4.0$	0.0020	14.0	1.1
	$4.0 < M_V < 5.0$	0.0024	19.5	2.0
	$5.0 < M_V < 8.0$	0.0074	20.0	6.5
	$M_V > 8.0$	0.014	20.0	12.3
	white dwarfs	0.005	20.0	4.4
	brown dwarfs	0.008	20.0	6.2
	stellar halo	0.0001	100.0	0.6

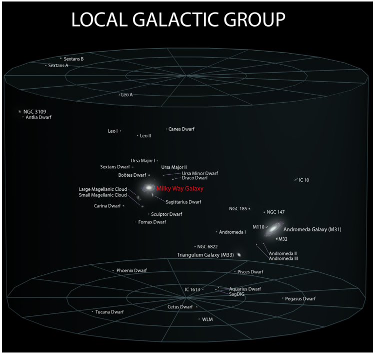
The Milky Way

We are also located is a peculiar region in the Milky Way : there is no ISM close to the Sun, we are in a bubble which is probably due to a past supernova event.



The Milky Way is not isolated !

It belongs to a group of galaxies called the local group that consists of two massive galaxies (the Milky Way and Andromeda) plus smaller galaxies.

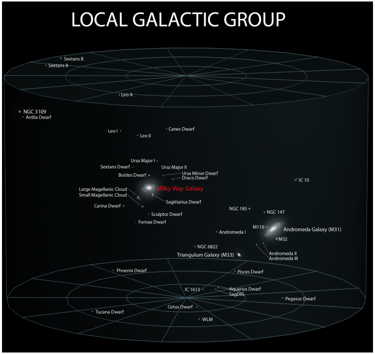


The local volume

It size is about 3 Mpc and it contains more than 54 galaxies (mostly dwarf spheroidals).

The local group consists of two smaller galaxy groups around its two main galaxies.

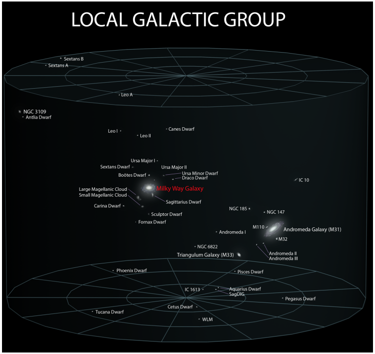
The two groups are separated by 0.8 Mpc and move towards each other at a velocity of 123 km/s.



The local volume

The two biggest satellite galaxies of the Milky are the Magellanic clouds (small and large).

They can be "seen" by naked eyes in the southern hemisphere.

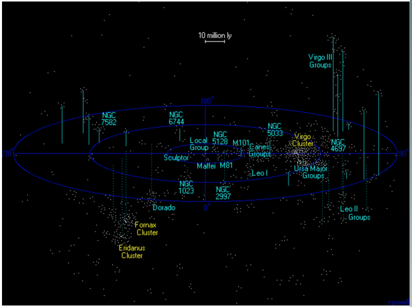


Beyond the local group

Larger structures than galaxy groups are called galaxy clusters.

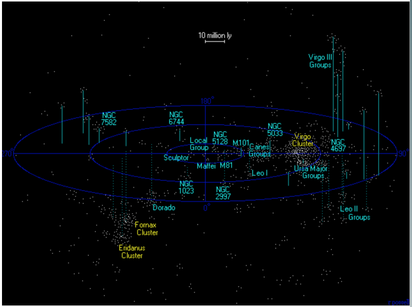
The closest and more studied are the Virgo cluster (@ 17 Mpc) and the Fornax cluster (@ 19 Mpc).

At these distances, one can study details of individual galaxies and one can start statistical studies.



Beyond the local group

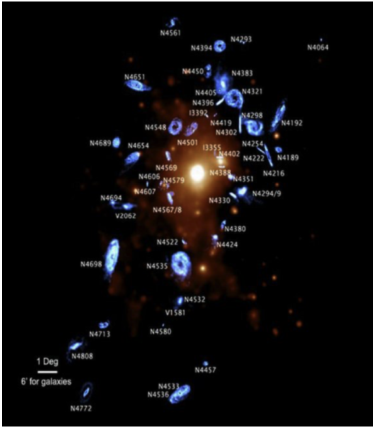
At 100 Mpc, there is another large, well studied cluster : Coma.



Beyond the local group

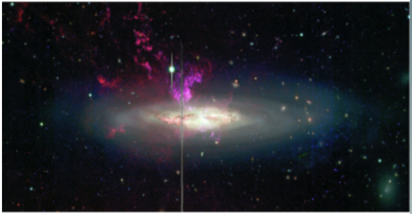
Comparing "clusters" and "field", one studies environmental effects on the evolution of galaxies.

HI image (21cm) of spiral galaxies [size $\times 10$], on top of a Virgo map in X rays $\Rightarrow$ disks are distorded and truncated

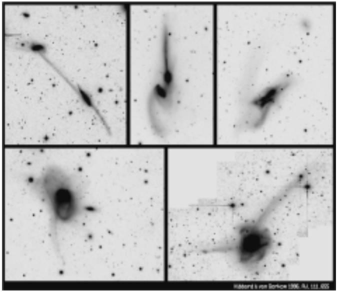


Comparing "clusters" and "field", one studies environmental effects on the evolution of galaxies.

optical image + Hα (656nm, recombination of ionized hydrogen) of a galaxy where the dynamical ram pressure stripping strips gaz from the galaxy



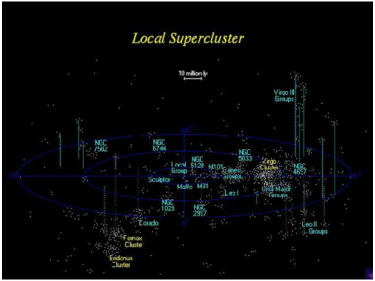
Beyond the local group



It is also in these environments that galaxies encounter violent processes such as galaxy fusion.

Structure larger that galaxy clusters are called superclusters.

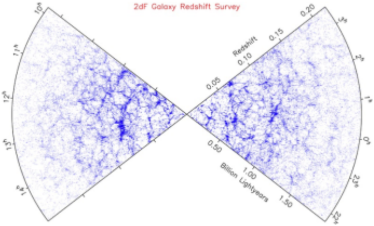
The Universe at larger scale



The universe at large scale reveals large-scale structures (clusters, filaments, sheets or walls, voids). One study these objects on a statistical basis. The expansion of the universe can not be neglected at these scales.

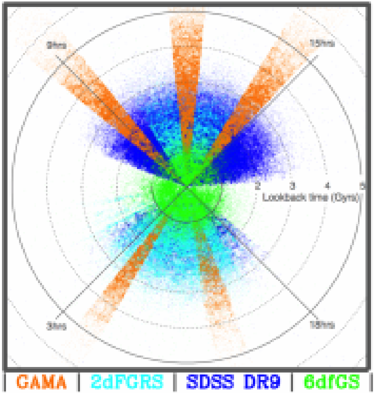
The Universe at larger scale

To study the universe at these large scale, one uses redshift surveys of galaxies (spectroscopic or photometric). An example 2dF redshift survey : 220,000 galaxies (1997-2002)
older surveys : CfA Redshift Surveys (1977-1990)
more recent : 6dFGS, SDSS (Sloan Digital Sky Survey), GAMA, VVDS



Map of the region covered by the recent spectroscopic surveys :

- GAMA : GAlaxy Mass Assembly DR2
- 2DFGRS : 2dF Galaxy Redshift Survey
- SDSS DR9 : Sloan Digital Sky Survey DR9
- 6dFGS : 6dF Galaxy Survey



Still many regions of the sky are not covered !

Redshift measurement

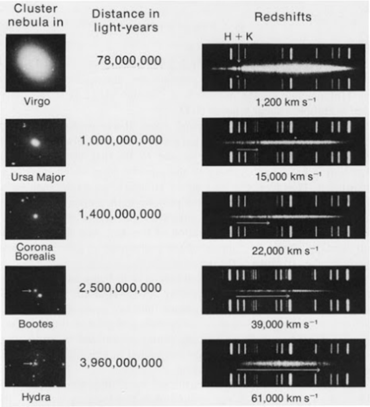
To estimate distances to distant galaxies, we use the fact that the universe is in expansion.

The velocity of the galaxies with respect to the observer induces a Doppler shifting of the spectral features towards the red (distant galaxies are moving away from us, the faster they move, the more distant they are).

A measure of the spectral shift gives then an estimate of the distance of the object if the cosmological model is known.

Redshift measurement

An example : redshift of the calcium H and K lines as a function of distance



Redshift measurement

The redshift is measured from the Doppler shift as

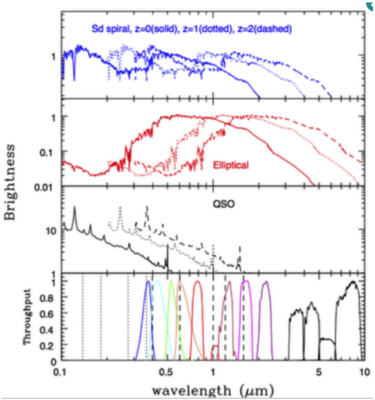
$$z \equiv \frac{\lambda_{obs} - \lambda_{em}}{\lambda_{em}} = \frac{v}{c}$$

where v is the recession velocity.
The last equality is valid only if $v \ll c$.

This definition is also sometime written as $1 + z \equiv \frac{\lambda_{obs}}{\lambda_{em}}$.

If the principle of measuring redshift is simple, the reality is more complex and spectroscopy is an expensive process. Hence one has to resort using photometry to measure redshifts. These redshifts are less accurate.

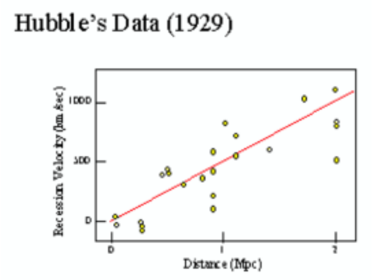
Also, one must take into account evolutionary effects !
Indeed, the further away a Galaxy is, the younger it will be due to constant nature of the speed of light. Hence these galaxies will be less evolved and it must be corrected for as their spectra are intrinsically a bit different from local galaxies.



The recession velocity of galaxies (corrected for their peculiar velocity and solar system velocity) is linked to their distance :

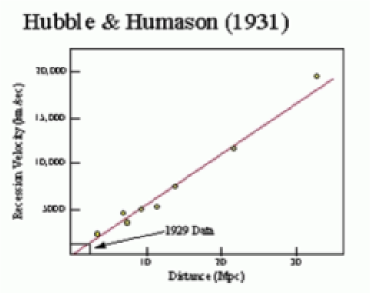
$$v_0 = H_0 d$$

with H_0 the Hubble's constant. It was first observed by Hubble in 1929.

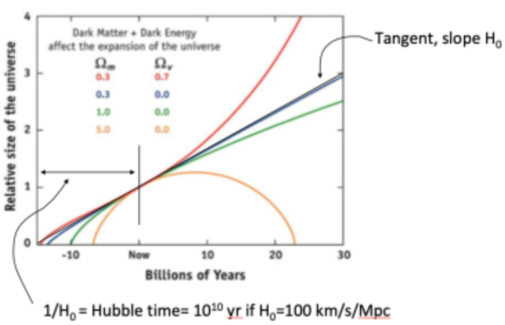


Elements of cosmology

Two years later, in 1931, he also showed that this linear relation is also valid at larger scale.



The value of H_0 is important to constrain cosmological models.



Elements of cosmology

The recession velocity is not only linked to distance but also to the age of the universe at a given redshift.

For objects whose recession velocity is not relativistic, we have

$$z \equiv \frac{\lambda_{obs} - \lambda_{em}}{\lambda_{em}} = \frac{v}{c}$$

The age of the universe at redshift z is given by

$$t(z) = \frac{1}{H_0} \int_{1+z}^{+\infty} \frac{\mathrm{d}x}{\sqrt{\Omega x^5 + \lambda_0 x^2 - (\Omega + \lambda_0 - 2)\, x^4}}$$

where $x = 1 + z$.

Elements of cosmology

Hence

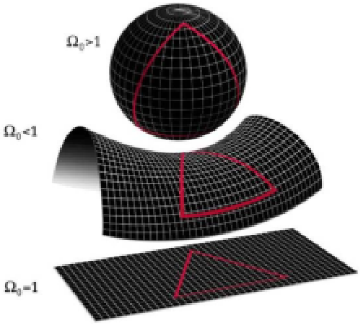
- at redshift z=10, the universe was only a few percent of its current age
- at redshift of z∼1, the universe was about half its current age.

More details on conventions and notations will be given in the lecture by D. Aubert. See also lecture of Y. Mellier in "Grands relevés et observatoires virtuels, Goutelas 2001", or lecture of J.Peacock (link).

Elements of cosmology

The geometry and shape of the universe is fixed by the cosmological parameters.

Three main shape exist : closed, open and flat



Main cosmological parameters :

- Hubble constant H_0 (also used $H_0 = h.100$ km/s/Mpc).
  - For each value of H_0, there exists a critical density ρ_{crit} for which the geometry of the Universe is Euclidian.
  - ρ_{crit} is given as a mass per unit volume, but it represents an energy density (from $E = mc^2$), and standard matter only contributes for a fraction of it.
  - If $h \sim 0.7$ then $\rho_{crit} \sim 2\ 10^{-29}\ h^2.g.cm^{-3}$

Elements of cosmology

Main cosmological parameters :

- Total density parameter $\Omega = \rho / \rho_{crit}$.
  - Equals 1 for a flat universe (favored model today), < 1 for an open Universe, > 1 for a closed Universe.
  - Ω is actually the sum of three separate components : $\Omega = \Omega_M + \Omega_\Lambda + \Omega_R$ which correspond respectively to the contribution of the matter (barionic or not), of the vacuum (dark energy)and to the contribution of relativistic particles (negligible today).

Main cosmological parameters :

- Deceleration parameter q
  - measures the slow down of the expansion velocity of the Universe (using the second derivative of the scale factor of the Universe). It fixes the past and future evolution of the distance between objects in the Universe. One writing of q : $q = \frac{1}{2}\Omega_M - \Omega_\Lambda$
  - Using supernovae, the measurement of the deceleration parameter gives $q \sim -0.55$, which (if $\Omega = 1$) gives $\Omega_M \simeq 0.3$ and $\Omega_\Lambda \simeq 0.7 (\Omega_R \simeq 8\,10^{-5})$.
  - q **is negative : expansion is accelerating.**

Introduction to stellar evolution

Hydrostatic equilibrium
Stellar energy sources
Modeling stellar structure
Stellar evolution

The idealized case

We will consider the idealized case of a spherically symmetric star, non-rotating, non-magnetic, single... a star on which no net force is acting.

Internal motions, such as convection, may occur but they are assumed to average out overall.

The star is supposed to be in hydrostatic equilibrium, in this case gravitational acceleration is compensated for at each point by the pressure gradient.

The idealized case

One considers a volume element of the star defined by its spherical surface d^2S and thickness $\mathrm{d}r$.
The hydrostatic equilibrium writes

$$-G\frac{\rho(r).M_r\,\mathrm{d}^2S\,\mathrm{d}r}{r^2}+P(r)\,\mathrm{d}^2S-P(r+\mathrm{d}r)\,\mathrm{d}^2S=0,$$

where the first term is the inwardly directed gravitational force on the volume.
Eliminating d^2S and noting that the pressure gradient is

$$P(r+\mathrm{d}r)-P(r)=\frac{\mathrm{d}P}{\mathrm{d}r}.\mathrm{d}r,$$

we obtain

$$\frac{\mathrm{d}P}{\mathrm{d}r}(r)=-G\frac{\rho(r).M_r}{r^2}\ \text{with}\ M_r=\int_0^r\rho(r).4\pi r^2\mathrm{d}r.$$

The idealized case

That is, equilibrium implies that the pressure gradient $\frac{\mathrm{d}P}{\mathrm{d}r} < 0$.

We have $M_{(r+\mathrm{d}r)} - M_r = \mathrm{d}M_r$ and the mass of a spherical shell of thickness $\mathrm{d}r$ within r and $r + \mathrm{d}r$ is given by

$$\mathrm{d}M_r = 4\pi r^2 \rho(r)\mathrm{d}r.$$

The idealized case

To solve the previous equation, we have two possible choices for the independent variable :

- r : Eulerian approach
- M_r : Lagrangian approach in which case the hydrostatic equilibrium writes $\frac{\mathrm{d}P}{\mathrm{d}M_r} = -\frac{GM_r}{4\pi r^4}$.

The idealized case

Notes :

- The hydrostatic equation above can also be obtained using a non-local energy principle (see section 1.2 of Hansen & Kawaler).
- Sometime, pressure P is a function of only $\rho(r)$. If this is realized, previous equations completely define the stellar structure.
  - It is the case of white dwarfs (degenerate electron gas). In this case the equation of state is given by $P \propto \rho^{5/3}$ for the non relativist case (low mass objects) or by $P \propto \rho^{4/3}$ in the relativistic case (mass close to the Chandrasekhar limit) or any polytropic model which are used for quick studies of internal structure.
- Pressure in the equations of stellar structure includes gas pressure but also radiation pressure

Energy generation, transport and thermal balance

If the energy generated in a given volume of a star is not transferred elsewhere, a non-equilibrium condition holds and the material heats up. On the other hand, if energy is removed as quickly as it is generated and not faster, then the material is said to be in thermal balance.

To express thermal balance quantitatively, consider a spherically symmetric shell of mass $\mathrm{d}M_r$ and thickness $\mathrm{d}r$. Within that shell we denote ϵ the power generated per gram (unit erg $g^{-1}\, s^{-1}$) also referred to as energy generation rate.

The total power generated in the shell is given by

$$4\pi r^2 \rho \epsilon \mathrm{d}r = \epsilon \mathrm{d}M_r.$$

To balance the power generated, there must be a net flux of energy leaving the shell. If $F(r)$ is the flux (units $\mathrm{erg\,cm^{-2}}\,s^{-1}$) with positive values implying a radially outwards flow, then $\mathscr{L}_r = 4\pi r^2 F(r)$ is the total power, or luminosity, in $\mathrm{erg}\,s^{-1}$, entering (or leaving) the shell 's inner face.

$\mathscr{L}_{r+\mathrm{d}r} = 4\pi r^2 F(r+\mathrm{d}r)$ is the luminosity leaving the outer face.

The difference between the two terms is the net loss or gain of power for the shell.

For thermal balance, that difference must equal the total power generated within the shell. Hence

$$\mathscr{L}_{r+\mathrm{d}r} - \mathscr{L}_r = \mathrm{d}\mathscr{L}_r = 4\pi r^2 \rho\epsilon \mathrm{d}r$$

$$\Rightarrow \frac{\mathrm{d}\mathscr{L}_r}{\mathrm{d}r} = 4\pi r^2 \rho\epsilon,$$

or in Lagrangian form

$$\frac{\mathrm{d}\mathscr{L}_r}{\mathrm{d}M_r} = \epsilon.$$

Stellar energy sources

There are two sources of energy in a star :

- gravitational
- thermonuclear

Gravitational energy source :

This source of energy is important when the star is contracting. In the case of equilibrium, no energy is generated and we will neglect this part.

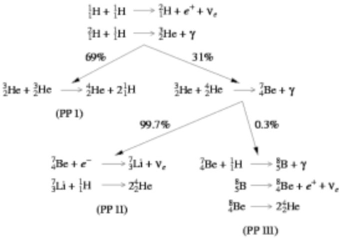
Thermonuclear energy source :

Nuclear reactions are the dominant energy sources in stars. The two main cycles that take place in stars are the pp-chain, the CNO cycle. These are hydrogen burning chain that produce ^4He from ^1H.

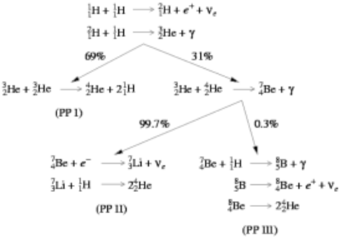
These are the dominant processes in what we call the main sequence and stars pass most of their lifetime being fueled by these processes

The pp-chain

This is the chain that occurs in low mass stars ($M < 1.1 M_\odot$) during the main phase of evolution. The chain of reaction is given by

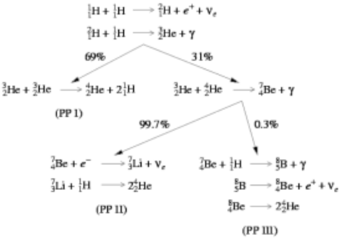


The pp-chain



- The PP1 chain dominates when the core temperature is 10-14 MK.
- The PP2 chain dominates in the range of temperatures 14-23 MK.
- The PP3 chain dominates above 23 MK.

The pp-chain



- The first reaction is very slow, its probability of occurring make it impossible to reproduce in laboratory but the densities in the core of stars is high enough so that they happen.
- Electron neutrinos production by this cycle $\Rightarrow$ solar neutrino problem.

The CNO-cycle

This is the main cycle for high mass stars (above $1.1 M_\odot$). Here carbon, nitrogen and oxygen are catalysts and contrary to the name of the cycle, it is a hydrogen burning sequence to produce ^4He.

$$
\begin{aligned}
C_6^{12} + H_1^1 &\rightarrow N_7^{13} + \gamma_0^0 \\
N_7^{13} &\rightarrow C_6^{13} + \bar{e}_1^0 + \nu_0^0 \\
C_6^{13} + H_1^1 &\rightarrow N_7^{14} + \gamma_0^0 \\
N_7^{14} + H_1^1 &\rightarrow O_8^{15} + \gamma_0^0 \\
O_8^{15} &\rightarrow N_7^{15} + \bar{e}_1^0 + \nu_0^0 \\
N_7^{15} + H_1^1 &\rightarrow C_6^{12} + He_2^4
\end{aligned}
$$

The CNO-cycle

$$C_6^{12} + H_1^1 \rightarrow N_7^{13} + \gamma_0^0$$
$$N_7^{13} \rightarrow C_6^{13} + \bar{e}_1^0 + \nu_0^0$$
$$C_6^{13} + H_1^1 \rightarrow N_7^{14} + \gamma_0^0$$
$$N_7^{14} + H_1^1 \rightarrow O_8^{15} + \gamma_0^0$$
$$O_8^{15} \rightarrow N_7^{15} + \bar{e}_1^0 + \nu_0^0$$
$$N_7^{15} + H_1^1 \rightarrow C_6^{12} + He_2^4$$

- The phase $N_7^{14} + H_1^1 \rightarrow O_8^{15} + \gamma_0^0$ is the slowest reaction.
- Nitrogen then accumulates in the core of the star.
- Neutrinos are generated in two reactions : detection possible.

triple-α process

When stars stop burning hydrogen (hydrogen is depleted in the core and in the shell around the core), helium is the next element to undergo nuclear fusion as it is present in high density in the stellar core and temperatures are high enough. At this stage, the star is on the giant's branch of evolution. Helium burning happens via the triple-alpha process which is a three body reaction.

$$^{4}\mathrm{He} + ^{4}\mathrm{He} \rightleftarrows ^{8}\mathrm{Be}$$
$$^{8}\mathrm{Be} + ^{4}\mathrm{He} \rightarrow ^{12}\mathrm{C} + \gamma$$

α capture

When the core's temperature increases and reaches 10^8K, other reactions start involving helium that creates heavier elements by capture of the helium nuclei :

$$
\begin{array}{ll}
{}^{12}_{6}\mathrm{C} + {}^{4}_{2}\mathrm{He} \longrightarrow {}^{16}_{8}\mathrm{O} + \gamma & E = 7.16\ \mathrm{MeV} \\
{}^{16}_{8}\mathrm{O} + {}^{4}_{2}\mathrm{He} \longrightarrow {}^{20}_{10}\mathrm{Ne} + \gamma & E = 4.73\ \mathrm{MeV} \\
{}^{20}_{10}\mathrm{Ne} + {}^{4}_{2}\mathrm{He} \longrightarrow {}^{24}_{12}\mathrm{Mg} + \gamma & E = 9.32\ \mathrm{MeV} \\
{}^{24}_{12}\mathrm{Mg} + {}^{4}_{2}\mathrm{He} \longrightarrow {}^{28}_{14}\mathrm{Si} + \gamma & E = 9.98\ \mathrm{MeV} \\
{}^{28}_{14}\mathrm{Si} + {}^{4}_{2}\mathrm{He} \longrightarrow {}^{32}_{16}\mathrm{S} + \gamma & E = 6.95\ \mathrm{MeV} \\
{}^{32}_{16}\mathrm{S} + {}^{4}_{2}\mathrm{He} \longrightarrow {}^{36}_{18}\mathrm{Ar} + \gamma & E = 6.64\ \mathrm{MeV} \\
{}^{36}_{18}\mathrm{Ar} + {}^{4}_{2}\mathrm{He} \longrightarrow {}^{40}_{20}\mathrm{Ca} + \gamma & E = 7.04\ \mathrm{MeV} \\
{}^{40}_{20}\mathrm{Ca} + {}^{4}_{2}\mathrm{He} \longrightarrow {}^{44}_{22}\mathrm{Ti} + \gamma & E = 5.13\ \mathrm{MeV} \\
{}^{44}_{22}\mathrm{Ti} + {}^{4}_{2}\mathrm{He} \longrightarrow {}^{48}_{24}\mathrm{Cr} + \gamma & E = 7.70\ \mathrm{MeV} \\
{}^{48}_{24}\mathrm{Cr} + {}^{4}_{2}\mathrm{He} \longrightarrow {}^{52}_{26}\mathrm{Fe} + \gamma & E = 7.94\ \mathrm{MeV} \\
{}^{52}_{26}\mathrm{Fe} + {}^{4}_{2}\mathrm{He} \longrightarrow {}^{56}_{28}\mathrm{Ni} + \gamma & E = 8.00\ \mathrm{MeV}
\end{array}
$$

Carbon fusion

Above core temperature of 6.10^8K (if the star has sufficient mass, the Sun does not), carbon will start burning via these processes

$$
\begin{aligned}
{}^{12}\mathrm{C} + {}^{12}\mathrm{C} &\rightarrow {}^{16}\mathrm{O} + 2\,{}^{4}\mathrm{He} \\
&\rightarrow {}^{20}\mathrm{Ne} + {}^{4}\mathrm{He} \\
&\rightarrow {}^{13}\mathrm{Na} + p^{+} \\
&\rightarrow {}^{23}\mathrm{Mg} + n \\
&\rightarrow {}^{24}\mathrm{Mg} + \gamma
\end{aligned}
$$

Oxygen fusion

and above 10^9K, it is the turn of oxygen

$$
\begin{aligned}
{}^{16}\mathrm{O} + {}^{16}\mathrm{O} &\rightarrow {}^{14}\mathrm{Mg} + 2\,{}^{4}\mathrm{He} \\
&\rightarrow {}^{28}\mathrm{Si} + {}^{4}\mathrm{He} \\
&\rightarrow {}^{31}\mathrm{P} + p^{+} \\
&\rightarrow {}^{31}\mathrm{S} + n \\
&\rightarrow {}^{32}\mathrm{S} + \gamma
\end{aligned}
$$

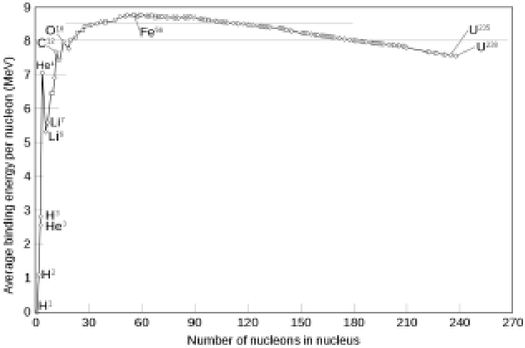
When does it stop ?

The reaction shown above release energy. Fusion reactions in the star will stop when energy will be required to continue the fusion processes instead of releasing energy.

When one looks at the binding energy per nucleon, we see that this happens when we reach the iron peak elements (Fe and Ni).

Above this mass, energy must be provided to continue the fusion process.

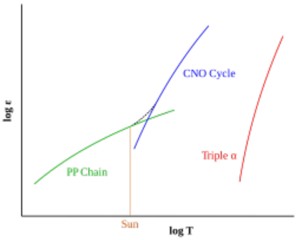
When does it stop ?



Nuclear process dominance region

Hence, at the end of their nuclear fusion time, the core of stars will be iron and Ni rich.

The dominance region of the various nuclear cycles can be summarized in this figure



Modeling stellar structure

Stellar structure model

To construct a stellar evolution model, one searches for a model that, given the total mass and the composition (as a function of radius or equivalent) returns the run of mass versus radius and the corresponding local values of pressure, density, temperature and luminosity. This needs microscopic physics implied in the following relations

- $P = P(\rho, T, \mathbf{X})$
- $E = E(\rho, T, \mathbf{X})$
- $\kappa = \kappa(\rho, T, \mathbf{X})$
- $\epsilon = \epsilon(\rho, T, \mathbf{X})$

where $\mathbf{X}$ is the composition of the star, κ the opacity and ϵ is the energy equation.

Stellar structure model

Differential relations (structural and thermal) have to be satisfied such as

- $\frac{\mathrm{d}P}{\mathrm{d}M_r} = -\frac{GM_r}{4\pi r^4}$
- $\frac{\mathrm{d}r}{\mathrm{d}M_r} = \frac{1}{4\pi r^2 \rho}$
- $\frac{\mathrm{d}\mathscr{L}_r}{\mathrm{d}M_r} = \epsilon$

or their Eulerian counterpart, as well as establish the mode of heat transfer (radiative or convective, based on the dell operator $\nabla = \frac{\mathrm{d}\ln T}{\mathrm{d}\ln P}$).

All this equations, when combined, are equivalent to a fourth-order differential equation in space or mass. It is a complicated problem to solve and a solution may not necessarily exist. If a solution exists it may not be unique.

Stellar structure model

For example, the figure below presents the radiative and convective zone as a function of the stellar mass for main sequence stars (where H burning takes place).

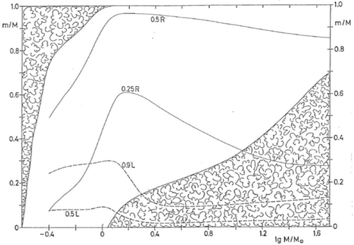


Fig. 22.7. The mass values m from centre to surface are plotted against the stellar mass M for the same zero-age main-sequence models as in Fig. 22.1. "Cloudy" areas indicate the extension of convective zones inside the models. Two solid lines give the m values at which r is 1/4 and 1/2 of the total radius R. The dashed lines show the mass elements inside which 50% and 90% of the total luminosity L are produced

Constructing a stellar evolution model is a complex business and more information will be given in the stellar physics lectures.

Stellar evolution and simple stellar populations

There are roughly speaking 7 main stages of stellar evolution which are summarized in the HR-diagram :

1. pre-main sequence (PMS) : this is the phase on contraction of the gas cloud. No nuclear reaction take place, heating is due to gravitational collapse

2 main sequence(MS) : Hydrogen burning phase in the core of the star. This is the longest stage of stellar evolution. The start of this phase is called the zero-age main sequence (ZAMS). The end of the main sequence is characterized by the turn-off where the stars start ascending the giant branch in the HR diagram (see below). Lifetime on this stage depends on the initial mass $t_{MS} \sim \left(\frac{M}{M_\odot}\right)^{-2.5} 10^{10} yr$.

Lifetimes of Main-Sequence Stars

Spectral Type	Surface Temperature (K)	Mass (Mass of Sun = 1)	Lifetime on Main Sequence (years)
O5	54,000	40	1 million
B0	29,200	16	10 million
A0	9600	3.3	500 million
F0	7350	1.7	2.7 billion
G0	6050	1.1	9 billion
K0	5240	0.8	14 billion
M0	3750	0.4	200 billion

Phases of stellar evolution

3 sub-giant phase : shell hydrogen burning phase, the luminosity of the star stays ∼ constant. Helium core grows. This phase duration ranges from a few million to one or two billion years.
4 red-giant phase (RGB) : shell-hydrogen burning phase. Helium core continue to grow. The convective outer layers of the star expands and luminosity increases.
5 horizontal branch : helium burning phase. After helium burning starts at the end of the RGB, the star contracts and increase its surface temperature until it reaches the instability strip (the horizontal branch)
6 Asymptotic giant branch (AGB) : H and He burning in shell around a carbon and oxygen hot core
7 post-AGB phase : nuclear reaction stop, the core of the star contracts to form a white dwarfs. Outer layers go through a TP-AGB superwind phase and create planetary nebula.

Phases of stellar evolution

The exact details on which phase a star will go through will depend on its mass on the main sequence.

Some examples on the duration of the various phases are given in the figure on the right.

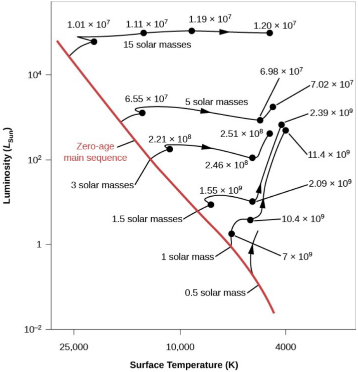
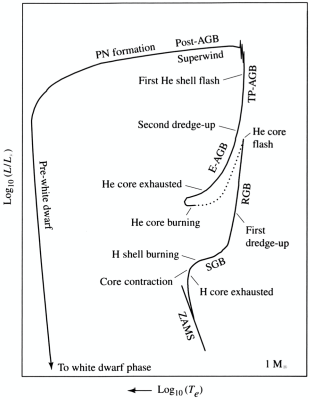
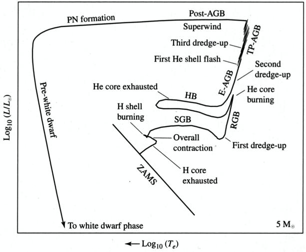


Figure 4. Evolutionary Tracks of Stars of Different Masses: The solid black lines show the predicted evolution from the main sequence through the red giant or supergiant stage on the H–R diagram. Each track is labeled with the mass of the star it is describing. The numbers show how many years each star takes to become a giant after leaving the main sequence. The red line is the zero-age main sequence.

Phases of stellar evolution

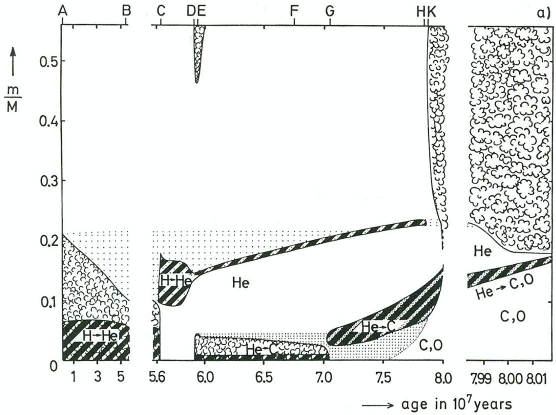
The exact shape of the track a star makes in the temperature-luminosity diagram (so called HR-diagram) depends on the mass. Here are two examples for a 1 and 5 solar mass star





Phases of stellar evolution

Note that the internal structure of the star evolves during the lifetime ! Here is an example for a $5M_\odot$ star.



Stellar populations

A stellar population is a group of stars whose age and chemical composition are alike :

- the stars are formed in the same phase of star formation,
- the stars are formed in a region where the interstellar medium is almost homogeneous.

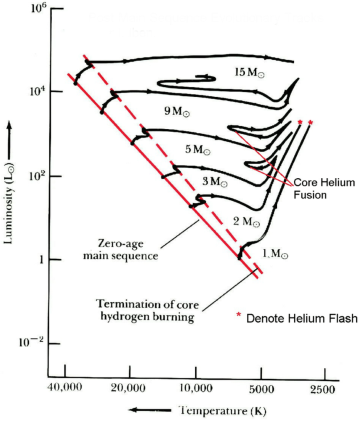
Stars evolve with time. They change

- temperature
- luminosity
- chemical composition (after the AGB)

Stellar populations

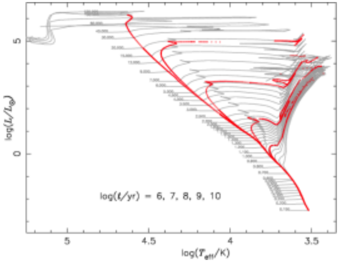
Hence the magnitude and color of star change and the properties of the stellar population will change with time (or more precisely with age).

Evolution depends on the mass on the ZAMS and on metallicity. The lifetime on the main sequence also depends on mass.



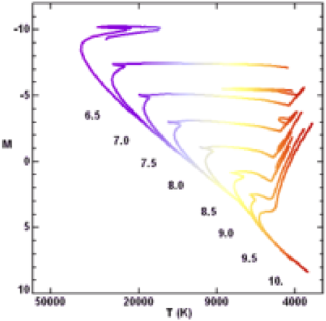
Stellar populations

As in a (simple) stellar population all stars have the same age, the track of a simple stellar population (SSP) in the HR diagram can be obtained by interpolating the evolutionary track of stars of different mass. This is called an isochrone.



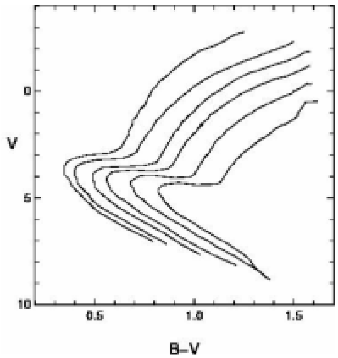
Effect of age

As the SSP ages, it comes redder as the massive blue stars progressively disappear, evolving towards the RGD and AGB. The turn-off point shifts towards low luminosities.



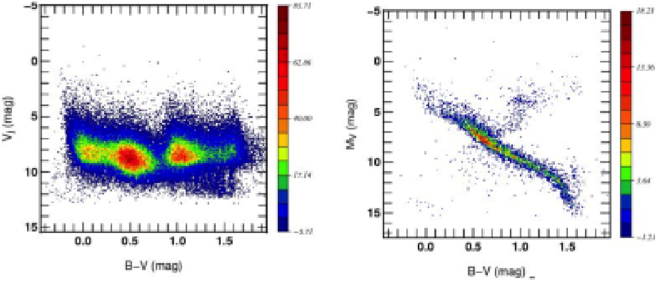
Effect of metallicity

Color-magnitude diagram (CMD) of SSPs of the same age, metallicity in increasing from left to right.



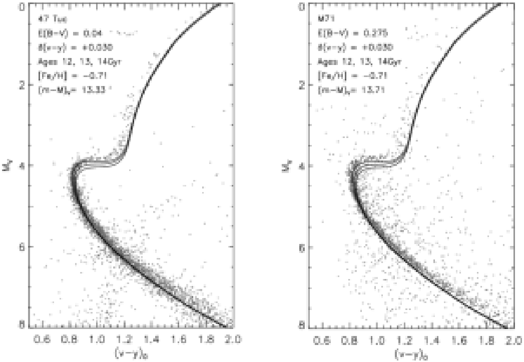
Effect of distance

Left : color apparent magnitude diagram for stars in the solar neighborhood. Right : for stars with known parallaxes, the color-absolute magnitude diagram.



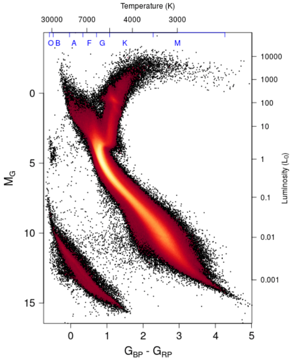
Example of SSP

Globular clusters (all the stars are at the same distance, the different evolutionary branches are easy to identify)



Superposition of SSPs

The case of Gaia CMD



Superposition of SSPs

One can go even further and try to separate the different populations.

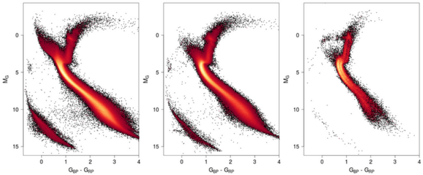
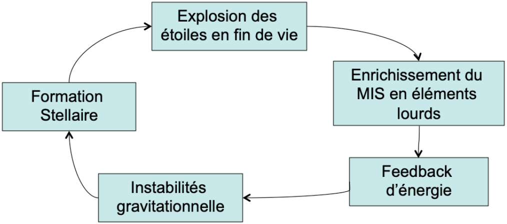


Fig. 21. *Gaia* HRDs with kinematic selections based on the tangential velocity: *panel a*: $V_T < 40\,\mathrm{km\,s^{-1}}$ (1 893 677 stars), *panel b*: $60 < V_T < 150\,\mathrm{km\,s^{-1}}$ (1 303 558 stars), and *panel c*: $V_T > 200\,\mathrm{km\,s^{-1}}$ (64 727 stars).

And there are surprises, for halo like kinematic, we see two populations appearing ! Lots of information on the evolution of a galaxy or cluster can be obtained by the analysis of CMDs and HR-diagram.

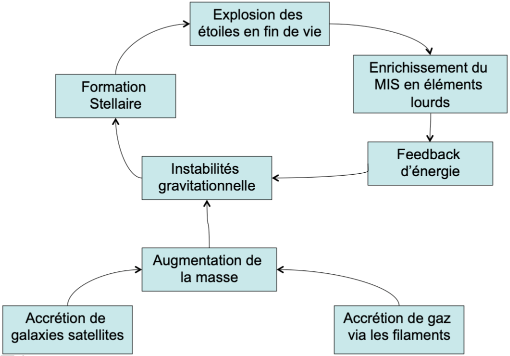
Introduction to chemical evolution

Cycle of matter



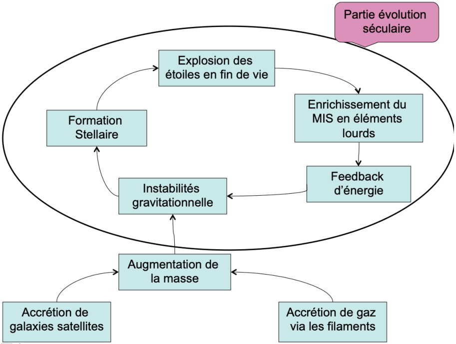
The evolution of galaxies is complex : processes take place both at large scales (macrophysics like gravitation) and small scales (heavy elements production)

Cycle of matter



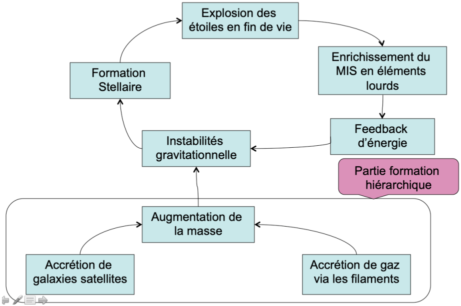
timescales of the different processes also quite different
$\Rightarrow$ full modeling not possible.

Cycle of matter



- Phases internal to the galaxy belong to the secular evolution (assuming other aspects can be neglected).

Cycle of matter



- Those parts depend of the environment of the galaxy and to its accretion history : link to hierarchical formation scenario (cosmology).

Notation

Astrophysics : use of metallicty Z.
⚠Metallicity does not really mean metals !

Metals = elements heavier than helium (and sometimes hydrogen) .

Metallicity is a mass ratio : ratio between the mass of heavy elements to the total mass of the gas.

Notation

$$Z = \frac{M_h}{M_g}$$

where M_h is the mass of heavy elements and M_g is the mass of the gas.

Example : Sun $Z_\odot = 0.02$.
We will use m for individual star masses and M for the mass of components (gas, stars, heavy elements...)

Closed box evolution

The closed box model : model build on two strong assumptions

- instantaneous recycling (no delay between star formation and the return of heavy elements in the ISM)
- instantaneous mixing (heavy elements are homogeneously distributed in the box)

Closed box evolution

Hidden assumption : gas accretion (or any interaction) from outside the box can be neglected over the time interval of the study.

Initial condition : at $t = 0$ all the matter is gas $M_h = 0$.

Closed box evolution

The variation of the metal content simply writes

$$\delta M_h = p\delta M_s - Z\,\delta M_s = (p - Z)\,\delta M_s$$

First term on the right : amount of metals formed by the generation of stars of total mass M_s.

p is called the yield, fraction of metals returned in the ISM by the stars (given by stellar physics models).

Second term is the amount of metals needed to produce the generation of stars.

Closed box evolution

One can deduce the evolution of the metallicity Z

$$\delta Z = \delta \left(\frac{M_h}{M_g} \right) = \frac{1}{M_g} (\delta M_h - Z \delta M_g)$$

which combined to the previous equation and to the equation of mass conservation $(\delta M_s = -\delta M_g)$ allows us to simplify the relation :

$$\delta Z = -p \, \frac{\delta M_g}{M_g}$$

Closed box evolution

If p remains unchanged for each generation of stars, one can integrate the relation

$$\boxed{Z(t) = -p \ln\left[\frac{M_g(t)}{M_g(0)}\right] + cste.}$$

With the initial condition $M_h(0) = 0$, $cste = 0$ else $cste = Z(0)$.

The relation between metallicity Z and the gas fraction is logarithmic.

A quantity that can be compared to observations is the cumulative distribution $M_{star}[< Z(t)]$ which represents the mass of stars with $Z < Z(t)$.

This quantity is directly

$$M_{star}[< Z(t)] = M_s(t) = M_g(0) - M_g(t).$$

Hence

$$M_{star}[< Z(t)] = M_g(0)\left(1 - \exp\left(-\frac{Z(t) - Z(0)}{p}\right)\right).$$

Closed box evolution

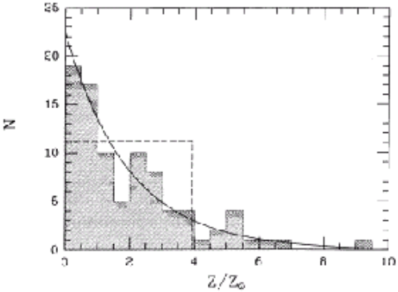
When all the gas is used

$$\boxed{\mathrm{d}M_{star}(Z) \propto \exp\left(-\frac{Z - Z(0)}{p}\right)\mathrm{d}Z}$$

where $\mathrm{d}M_{star}(Z)$ is the mass of stars with $Z \in [Z, Z + \mathrm{d}Z]$.

Closed box evolution

This result is extremely simplified and hides many details of the physics behind. However, it can give informations on the evolution of certain populations (eg bulges)



Rich (1990)

FIG. 8.—Differential abundance distribution of bulge giants compared to two limiting cases of the simple model of chemical evolution. *Solid line*: simple "closed box" model with complete gas consumption; $\langle z \rangle = 2.0 z/z_{\odot}$. *Dashed line*: Simple model, in the limiting case where a small fraction of the initial volume of gas is converted to stars, the remainder being lost from the system.

Complete model

Solving the full problem is much more complicated and requires usually numerical methods.

One can however review the different ingredients and equations needed to model the chemical evolution of a galaxy (or region of a galaxy).

Complete model

Conservation laws :

$$M = M_s + M_g \quad (2)$$

$$\frac{dM}{dt} = f - e \quad (3)$$

$$\frac{dM_s}{dt} = \Psi - E \quad (4)$$

$$\frac{dM_g}{dt} = -\Psi + E + f - e \quad (5)$$

$$= \frac{dM}{dt} - \frac{dM_s}{dt}$$

M = total mass

M_s = stellar mass

M_g = gas mass

f = infall rate

e = outflow rate

Ψ = star formation rate

E = stellar mass loss rate

Complete model

E is given by :

$$E(t) = \int_{m_t}^{\infty} [m - \omega_m] \Psi(t - \tau_{MS}(m)) \Phi(m) \,\mathrm{d}m \qquad (6)$$

this integration is over the mass of stars (on the ZAMS).

- $m_t \rightarrow$ turnoff mass at time t
- $m - \omega_m \rightarrow$ mass of the outflow ($m =$ initial ZAMS mass and $\omega_m =$ remnant mass)
- $\tau_m(m) \rightarrow$ main sequence lifetime for a star of mass m on the ZAMS
- $\Psi(t - \tau_{MS}(m))\Phi(m) \rightarrow$ formation rate of stars of mass m at time $t - \tau_{MS}(m)$ = death rate at t

Complete model

Equation (6) gives the total mass of gas in outflow. We must also know the outflow mass of heavy elements. For metals, equation (5) rewrites

$$\frac{\mathrm{d}Z\,M_g}{\mathrm{d}t} = -Z\,\Psi + E_Z + Z_f.f - Z\,e \tag{7}$$

with $Z_f.f$ = infall of metals and E_z the outflow rate of metals given by

$$E_z(t) = \int_{m_t}^{\infty} [(m-\omega_m)Z(t-\tau_{MS}(m)) + m.p_{Z_m}]\Psi(t-\tau_{MS}(m))\Phi(m)\mathrm{d}m. \tag{8}$$

Here $m.p_{Z_m}$ gives the heavy elements produced by a star of mass m born at a metallicity Z.

Complete model

To solve these equations, we need to know the details of stellar evolution a a function of mass m

+ star formation history
+ initial mass function (IMF).

In a real galaxy, star formation is localized spatially and instabilities generate large scale motions of the gas and stars (blurring and churning). This model hence misses the dynamical effects.

Complete model

One can simplify this equations to recover the closed box approximation if one considers the instantaneous mixing approximation. To simplify the notations, we can define

$$R = \int_{m1}^{\infty} (m - \omega_m)\Phi(m)\mathrm{d}m \tag{9}$$

the mass returned to the ISM by generated stellar mass and

$$y = \frac{1}{1-R}\int_{m1}^{\infty} m.p(\tau_{MS})\Phi(m)\mathrm{d}m \tag{10}$$

which is a yield that gives the mass of metals produced by remnant mass. $m1$ is the turnoff mass at time t of a stellar population generated at t_1. NB : integrals (9) and (10) are over dead stars.

Complete model

With these definitions $(1-R)M$ is the mass of remnants and the ejected gas is given by $R.M$.

If we add the instantaneous recycling approximation, then $\Psi(t-\tau_{MS}) \approx \Psi(t)$ and if the IMF does not depend on t (e.g. $R = cste$) then
(6) $\Rightarrow E(t) = R.\Psi(t)$
(8) $\Rightarrow E_Z(t) = R.Z(t)\Psi(t) + (1-R)y(t)\Psi(t)$

(7) then becomes $\frac{\mathrm{d}Z.M_g}{\mathrm{d}t} = (1-R)(-Z+y)\Psi + Z_f.f - Z.e$

Complete model

Now combine (6) and (4) to get

$$\frac{\mathrm{d}M_s}{\mathrm{d}t} = (1-R)\Psi(t)$$

Then (6) and (5)

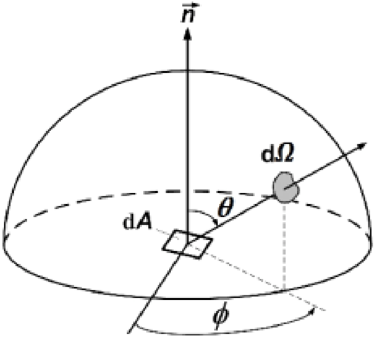
$$\frac{\mathrm{d}M_g}{\mathrm{d}t} = -(1-R)\Psi(t) + f - e$$

$+(8) \Rightarrow M_g \frac{\mathrm{d}Z}{\mathrm{d}t} = (1-R)\,y(t)\Psi(t) + (Z_f - Z)\,f + e.Z.$
which is the closed box approximation as $f = e = 0$ (given the appropriate initial conditions).

Introduction to stellar atmospheres and radiation transfer

Geometry

We will consider the flow of energy in a solid angle $\mathrm{d}\Omega$, emitted by a unit area $\mathrm{d}A$ whose normal direction is given by the unit vector $\vec{n}$, towards the direction of the observer defined by $\vec{i}$.



Intensity (or specific intensity)

Intensity $I_\nu(\vec{r},\vec{i},t)$ is the flow of energy at a specific location in a specific direction, per unit time, per unit frequency ν, per unit solid angle around that direction and per unit area perpendicular to that direction.

Units :

$$\begin{aligned}[I_\nu] &= \mathrm{erg\,cm^{-2}\,s^{-1}\,H_z{}^{-1} sr^{-1}}\ \mathrm{(cgs)} \\ &= \mathrm{J\,m^{-2}\,s^{-1}\,H_z{}^{-1} sr^{-1}}\ \mathrm{(SI)}\end{aligned}$$

Intensity (or specific intensity)

The **ernergy per unit frequency** is then given by

$$\begin{aligned} \mathrm{d}E_\nu &= I_\nu(\vec{i}\cdot\vec{n})\,\mathrm{d}A\,\mathrm{d}t\,\mathrm{d}\nu\,\mathrm{d}\Omega \\ &= I_\nu\cos\theta\,\mathrm{d}A\,\mathrm{d}t\,\mathrm{d}\nu\,\mathrm{d}\Omega \end{aligned}$$

Note that along the beam, I_ν is constant with distance.

Mean intensity

The **mean intensity** is the directional average of the specific intensity, e.g.

$$J_\nu = \frac{\int I_\nu \mathrm{d}\Omega}{\int \mathrm{d}\Omega} = \frac{1}{4\pi}\int I_\nu \mathrm{d}\Omega$$
$$= \frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi} I_\nu \sin\theta \,\mathrm{d}\theta\,\mathrm{d}\phi$$

If the radiation is isotropic $J_\nu = I_\nu$.

Flux

The **flux** $F_\nu(\vec{r}, t)$ is the net flow of energy per unit time, per unit area and per unit frequency.

Units :

$$[F_\nu] = \text{erg}\,\text{s}^{-1}\,\text{cm}^{-2}\,\text{Hz}^{-1}$$

It relates to the intensity via $F_\nu = \int I_\nu(\vec{i} \cdot \vec{n})\mathrm{d}\Omega = \int I_\nu \cos\theta\,\mathrm{d}\Omega$.

Note :

- Isotropy $\Rightarrow \quad F_\nu = 0$
- $F_\nu \propto \frac{1}{r^2}$
- F_ν is really a vector.

Radiation pressure

The **radiation pressure** $P_\nu = \frac{1}{c} \int I_\nu \cos^2\theta \, \mathrm{d}\Omega$.

Units :

$$
\begin{aligned}
[P_\nu] &= \mathrm{dyne\,cm^{-2}\,Hz^{-1}} \ \ \mathrm{(cgs)} \\
&= \mathrm{N\,m^{-2}\,Hz^{-1}} = \mathrm{Pa\,Hz^{-1}} \ \ \mathrm{(SI)}
\end{aligned}
$$

Note : isotropy $\Rightarrow P_\nu$ is scalar (otherwise it is a second rank tensor).

Eddington moments (fluid approximation)

Define $\mu = \cos\theta$

$$J_\nu = \frac{1}{2}\int_{-1}^{+1} I_\nu \mathrm{d}\mu = \text{Mean Intensity, energy density}$$

$$H_\nu = \frac{1}{2}\int_{-1}^{+1} I_\nu\, \mu\, \mathrm{d}\mu \sim \text{Eddington flux, momentum density}$$

$$K_\nu = \frac{1}{2}\int_{-1}^{+1} I_\nu\, \mu^2 \mathrm{d}\mu \sim \text{Radiation Pressure}$$

$$\text{Eddington K} - \text{vector}$$

Note : these are only valid if I_ν is independent on ϕ.

WARNING

ALWAYS CHECK THE UNITS

Be aware that different authors use different notations ! You have to rely on the units to make sure of the quantity you are dealing with.

Radiative transfer

Radiative processes

Radiative transfer is the change in I_ν as the radiation propagates.

There are four processes that can influence I_ν :

1. scattering (directional change of radiation propagation or absorption followed immediately by emission)
   - Coherent or elastic : emission at the same frequency
   - Isotropic : radiates equally in all directions
2. Doppler or redshift (frequency change)

Radiative processes

There are four processes that can influence I_ν :

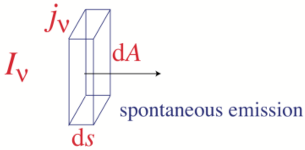
3 absorption : radiation absorbed by matter

4 emission :

- Spontaneous Emission : matter spontaneously emits a photon/radiation
- Stimulated Emission : passing radiation stimulates matter to emit in the same frequency and direction. Stimulated emission is mathematically the same as negative absorption - both proportional to the incoming radiation

All can be related to the matrix element for interaction - interrelated by Einstein relations

Spontaneous emission

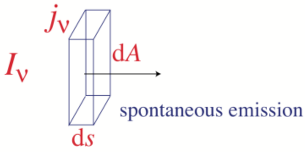


Given a source isotropically radiating power into a band $\mathrm{d}\nu$ at a rate $\frac{power}{volume} = P_\nu\,\mathrm{d}\nu$, the amount of energy radiated per unit solid angle is

$$\mathrm{d}E_{em} = P_\nu \mathrm{d}\nu\, \mathrm{d}V\, \mathrm{d}t \frac{\mathrm{d}\Omega}{4\pi}.$$

If one generalizes this relation to non-isotropic radiation, then $\frac{P_\nu}{4\pi} \rightarrow j_\nu(\Omega)$ which is called the monochromatic emission coefficient.

Spontaneous emission



This emission coefficient can also be given per unit mass, $\epsilon_\nu = 4\pi\, j_\nu/\rho$ where ρ is the mass density.

Then $\mathrm{d}E_{em} = \frac{\epsilon_\nu}{4\pi}\mathrm{d}\nu\,\mathrm{d}m\,\mathrm{d}t$ where $\mathrm{d}m = \rho\,\mathrm{d}V$.

Spontaneous emission

The effect of spontaneous emission on I_ν is then given by

$$\begin{aligned} \mathrm{d}E_{em} &= \mathrm{d}I_\nu \,\mathrm{d}\nu \,\mathrm{d}A \,\mathrm{d}t \,\mathrm{d}\Omega \\ &= j_\nu \,\mathrm{d}\nu \,\mathrm{d}V \,\mathrm{d}t \,\mathrm{d}\Omega \end{aligned}$$

where we identify $\mathrm{d}V = \mathrm{d}s\,\mathrm{d}A \Rightarrow \mathrm{d}I_\nu = j_\nu \,\mathrm{d}s$.

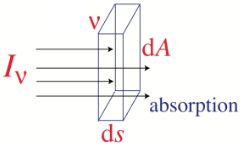
This leads to the radiative transfer equation in the case of spontaneous emission

$$\frac{\mathrm{d}I_\nu}{\mathrm{d}s} = j_\nu$$

whose solution is given by

$$I_\nu(s) = I_\nu(s_0) + \int_{s_0}^{s} j_\nu(s)\,\mathrm{d}s.$$

Absorption process



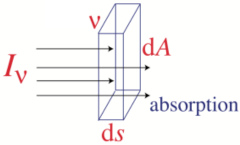
WARNING
ν in the figure must be understood as χ_ν in the text below.

Travelling through a material, a fraction χ_ν of the radiation is absorbed per unit length. The equation of radiative transfer in this case is written as

$$\frac{\mathrm{d}I_\nu}{I_\nu} = -\chi_\nu \mathrm{d}s$$

where χ_ν is called the absorption coefficient.

Absorption process



If the material does not emit radiation, the solution to this equation is an exponential suppression :

$$\ln I_\nu = -\int \chi_\nu \mathrm{d}s + C$$

$$\Rightarrow I_\nu(s) = I_\nu(s_0) \exp\left(-\int \chi_\nu \mathrm{d}s\right).$$

Absorption process

Absorption and cross-sections :

When one uses a particle description of absorbers, the absorption coefficient is related to the cross-section for interaction of the absorbers.

Given the number density n of absorbers, the covering fraction is given by

$$\mathrm{d}A_{abs} = \sigma\,\mathrm{d}N = \sigma\,n\,\mathrm{d}V = \sigma\,n\,\mathrm{d}A\,\mathrm{d}s.$$

Absorption process

The radiative transfer equation rewrites

$$\frac{\mathrm{d}I_\nu}{I_\nu} = -\sigma\, n\, \mathrm{d}s.$$

Hence $\chi_\nu = \sigma\, n$.

- Applies to a generalized version of σ as it can depend on frequency.
- The total distance a photon can travel is much greater than $s = \tau\, L$ as an individual photon propagates through the medium via scattering as a random walk which will be discussed later in this chapter.

Stimulated emission

The stimulated emission is identical to the absorption case but with $\chi_\nu < 0$.

The solution is then an exponential growth of I_ν along the path.

Radiative transfer equation (general case)

If one note $\mathrm{d}s$ the line element along the direction of propagation of the radiation, the radiative transfer equation can be written as

$$\frac{\mathrm{d}I_\nu}{\mathrm{d}s} = j_\nu - \chi_\nu I_\nu$$

where j_ν is the emission coefficient per unit volume and χ_ν is the extinction coefficient per unit distance. The extinction term with this notation contains both the absorption and scattering components.

Units :

$$[j_\nu] = \mathrm{erg\,cm^{-3}\,s^{-1}\,Hz^{-1}\,sr^{-1}}$$
$$[\chi_\nu] = \mathrm{cm^{-1}}$$

Radiative transfer equation (general case)

Note that the emission coefficient is related to the energy added to the beam, $\mathrm{d}E_\nu = j_\nu \mathrm{d}s\,\mathrm{d}A\,\mathrm{d}t\,\mathrm{d}\nu\,\mathrm{d}\Omega$.

Likewise, the extinction coefficient relates to the energy removed from the beam by $\mathrm{d}E_\nu = \chi_\nu\, I_\nu\,\mathrm{d}A\,\mathrm{d}t\,\mathrm{d}\nu\,\mathrm{d}\Omega$.

The removal of light (energy) is proportional to both the number of photons and to the number of gas particles.

Radiative transfer equation (general case)

The extinction coefficient can be written down in several ways :

$$\begin{aligned} \mathrm{d}I_\nu &= -\chi_\nu \, I_\nu \, \mathrm{d}s \\ &= -\alpha_\nu \, N \, I_\nu \, \mathrm{d}s \\ &= -\kappa_\nu \, \rho \, I_\nu \, \mathrm{d}s \end{aligned}$$

where N is the number density of absorbers ($[\mathrm{cm}^{-3}]$) and ρ the mass density ($[\mathrm{g\,cm}^{-3}]$).

Units : $[\alpha_\nu] = \mathrm{cm}^{-2}$ per absorber and $[\kappa_\nu] = \mathrm{cm}^{-2}\,\mathrm{g}^{-1}$.

κ_ν is often called opacity or mass extinction coefficient.

Radiative transfer equation (general case)

It is useful in radiative transfer to measure the length in units of the typical path length to interaction or absorption $L = \frac{1}{\chi_\nu}$.
The total path length in units of L then becomes

$$\int \chi_\nu \mathrm{d}s = \int \frac{\mathrm{d}s}{L} \equiv \tau_\nu$$

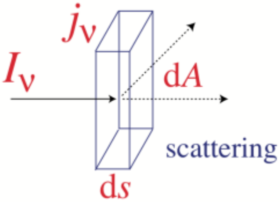
where τ_ν is called the optical depth.
The radiative transfer equation then writes

$$\frac{\mathrm{d}I_\nu}{\mathrm{d}s} = -\chi_\nu I_\nu + j_\nu$$

$$\Leftrightarrow \quad \frac{\mathrm{d}I_\nu}{\mathrm{d}\tau_\nu} = -I_\nu + S_\nu$$

where $S_\nu \equiv \frac{j_\nu}{\chi_\nu}$ is the source function.

Scattering



Scattering can be viewed as absorption AND emission where emission is directly proportional to absorption.

In the case of isotropic scattering, a fraction $\chi_\nu I_\nu$ of radiation is absorbed and re-radiated into 4π, hence

$$j_\nu = \chi_\nu \int \frac{\mathrm{d}\Omega}{4\pi} I_\nu = \chi_\nu J_\nu .$$

Scattering

Thus, the source function $S_\nu = j_\nu/\chi_\nu = J_\nu$ and, at high optical depth, the specific intensity approaches its angle averaged value J_ν.

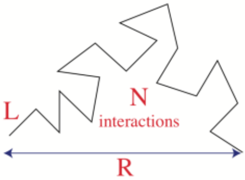
Despite this simple interpretation, the radiative transfer equation becomes an integro-differential equation as it depend on I_ν not only on the observation direction but on ALL the directions :

$$\begin{aligned} \frac{\mathrm{d}I_\nu}{\mathrm{d}\tau_\nu} &= -I_\nu + J_\nu \\ &= -I_\nu + \int \frac{\mathrm{d}\Omega}{4\pi} I_\nu \end{aligned}$$

.

Scattering

Scattering process can be thought of as a random walk as the scattering direction is random.
If one sketches the path a photon will follow in the medium one can define a few quantities



Assume the photon travels a distance L before being scattered and that it will interact N times before leaving the medium.

Scattering

One can write $\mathbf{R} = \mathbf{r}_1 + \mathbf{r}_2 + \ldots + \mathbf{r}_N$ where $\mathbf{r}_i = L\vec{\mathbf{r}}_i$ with $\vec{\mathbf{r}}_i$ the unit vector in the direction of propagation.

The total distance traveled by the photon is then $\langle R^2 \rangle = \langle \mathbf{R} \cdot \mathbf{R} \rangle = \langle \mathbf{r}_1 \cdot \mathbf{r}_1 \rangle \ldots + \langle \mathbf{r}_N \cdot \mathbf{r}_N \rangle + 2\langle \mathbf{r}_1 \cdot \mathbf{r}_2 \rangle \ldots$.

The cross terms being uncorrelated, we have $\langle \mathbf{r}_i \cdot \mathbf{r}_j \rangle = \delta_{ij} L^2$, therefor the average distance traveled is given by

$$\langle R^2 \rangle = N L^2 \text{ or } R_{rms} = \sqrt{N} L.$$

Scattering

How many interactions (scatterings) before escaping at distance R ?

- optically thick medium : $N = R^2/L^2 = \tau_\nu^2$
- optically thin case $(\tau_\nu \ll 1)$: a typical photon does not scatter and so by definition a fraction τ_ν will interact once, the rest zero and so the average $N = \tau_\nu$

A quick estimate is then given by $N = max(\tau_\nu, \tau_\nu^2)$.

Multiple processes

When dealing with multiple processes, the differential elements add together. For example, the total opacity is the sum of the individual opacities, so highest opacity process is most important for blocking the radiation I_ν.

Energy escapes in the spatial channel with lowest opacity !

- Example : a transition line vs continuum scattering ? photons will wander in frequency out of line and escape through lower opacity scattering → lines are often dark (sun)

Multiple processes

If one separates the scattering ($\chi_{\nu s}$) and absorption processes ($\chi_{\nu a}$), the radiative transfer equation write

$$\frac{\mathrm{d}I_\nu}{\mathrm{d}s} = -\chi_{\nu a}(I_\nu - S_{\nu a}) - \chi_{\nu s}(I_\nu - J_\nu)$$

or

$$\frac{\mathrm{d}I_\nu}{\mathrm{d}s} = -(\chi_{\nu a} + \chi_{\nu s})I_\nu + (\chi_{\nu a}S_{\nu a} + \chi_{\nu a}J_\nu)$$

which leads to combined absorption coefficient and source function given by $\chi_\nu = \chi_{\nu a} + \chi_{\nu s}$ and $S_\nu = \frac{j_\nu}{\chi_\nu} = \frac{\chi_{\nu a}S_{\nu a} + \chi_{\nu s}J_\nu}{\chi_{\nu a} + \chi_{\nu s}}$.

Multiple processes

- The typical length before absorption or scattering is then $L = 1/\chi_\nu = 1/(\chi_{\nu a} + \chi_{\nu s})$.
- The fraction of photons that ends in absorption is $\epsilon_\nu = \frac{\chi_{\nu a}}{\chi_{\nu a}+\chi_{\nu s}}$. Note that $1-\epsilon_\nu$ is the single scattering albedo.
- The source term can write $S_\nu = \epsilon_\nu S_{\nu a} + (1-\epsilon_\nu) J_\nu$.

Formal solution to the radiative transfer equation :

$$I_\nu(\tau_\nu) = I_\nu(0) e^{-\tau_\nu} + \int_0^{\tau_\nu} S_\nu(\tau'_\nu) e^{-(\tau_\nu - \tau'_\nu)} \mathrm{d}\tau'_\nu$$

Multiple processes

In the special case where S_ν is independent of τ_ν the solution becomes

$$\begin{aligned} I_\nu(\tau_\nu) &= I_\nu(0)e^{-\tau_\nu} + S_\nu \int_0^{\tau_\nu} e^{-(\tau_\nu - \tau'_\nu)} \mathrm{d}\tau'_\nu \\ &= I_\nu(0)e^{-\tau_\nu} + S_\nu(1 - e^{-\tau_\nu}) \end{aligned}$$

At low optical depth, I_ν is unchanged. At high optical depth, $I_\nu \rightarrow S_\nu$

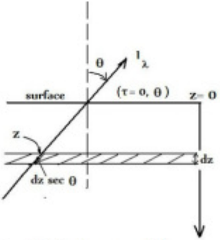
Warning

The integration is along the path of radiation (direction dependent), the source function can depend on I_ν in a different direction !

Stellar atmosphere models

Transfer equation in model atmospheres

Plane-parallel atmosphere : $\frac{\Delta R}{R_*} \sim 0$



Simplified plane-parallel model for stellar atmospheres

Let's note $\mu = \cos\theta$, then $ds = \frac{dz}{\cos\theta} = \frac{dz}{\mu}$.

The optical depth is defined as $d\tau_\nu = -\chi_\nu dz$ hence $\tau_\nu = \int_\infty^{z_0} d\tau_\nu$.

Transfer equation in model atmospheres

The radiative transfer equation states

$$I_\nu(s+\mathrm{d}s) = I_\nu(s) + j_\nu \mathrm{d}s - \chi_\nu I_\nu \mathrm{d}s = I_\nu(s) + \mathrm{d}I_\nu$$

$$\mathrm{d}I_\nu = j_\nu \mathrm{d}s - \chi_\nu I_\nu \mathrm{d}s \Rightarrow \frac{\mathrm{d}I_\nu}{\mathrm{d}s} = j_\nu - \chi_\nu I_\nu.$$

Using the definition of the source term $S_\nu = \frac{j_\nu}{\chi_\nu}$ and using the definition of the optical depth and the μ variable one gets the final equation

$$\mu \frac{\mathrm{d}I_\nu}{\mathrm{d}\tau_\nu} = I_\nu - S_\nu.$$

Extended atmospheres : $\frac{\Delta R}{R_*} \neq 0 \Rightarrow \frac{\partial \theta}{\partial r} \neq 0$

The transfer equation in that case becomes

$$\frac{\mu}{\kappa_\nu \rho} \frac{\partial I_\nu}{\partial r} - \frac{1-\mu^2}{\kappa_\nu \rho\, r} \frac{\partial I_\nu}{\partial \mu} = S_\nu - I_\nu .$$

- This equation is the one to use when stars have large photospheres (giants and supergiants)

When solving the transfer equation, we need to know the source function. We need to assume an equilibrium between matter and radiation.

Matter-radiation equilibrium

There are four situations for equilibrium between matter and radiation :

1. T.E. : Thermodynamic equilibrium (Boltzmann, Saha, Maxwell, Kirchhoff, Planck)
2. L.T.E : Local Thermodynamic Equilibrium (Boltzmann, Saha, Maxwell, Planck)
3. S.E : Statistical Equilibrium (Maxwell)
4. N.L.T.E : Non-LTE, absence of LTE

The case of matter

we have a Maxwell distribution of the velocities :

1D : $n(v_x) \propto T^{-1/2} \exp(-\frac{m v_x^2}{2kT})$

3D : $n(\mathbf{V}) \propto T^{-3/2}\mathbf{V}^2 \exp\left(-\frac{m\mathbf{V}^2}{2kT}\right)$

Saha-distribution for ionization population :

$$\frac{N_{r+1}}{N_r} = \frac{1}{N_e} 2 \frac{U_{r+1}}{U_r} \left(\frac{2\pi m_e kT}{h^2}\right)^{3/2} \exp\left(-\frac{\chi_r}{kT}\right).$$

Thermodynamic Equilibrium

Boltzmann distribution of excitation population :

$$\frac{n_{r,s}}{n_{r,t}} = \frac{g_{r,s}}{g_{r,t}} \exp\left(-\frac{\chi_{r,s} - \chi_{r,t}}{kT}\right)$$

with $s > t, g_{r,n}$ are the statistical weights of the excited states and $\chi_{r,n}$ the excitation potentials of the given state.

This can also be written as

$$\frac{n_{r,s}}{N_r} = \frac{g_{r,s}}{U_r(T)} \exp\left(-\frac{\chi_{r,s}}{kT}\right)$$

where $N_r = \sum_s n_{r,s}$ and $U_r(T) = \sum_s g_{r,s} \exp\left(-\frac{\chi_{r,s}}{kT}\right)$.

Thermodynamic Equilibrium

Case of the radiation :

All processes are in equilibrium with its inverse process → there is a detailed balance.

Equilibrium between the photons (radiation) and the gas (matter) implies that **only one temperature** describes both states (matter and radiation).

Thermodynamic Equilibrium

Also in TE intensity is isotropic ⇒ NO FLUX, then

$$\frac{\mathrm{d}I_\nu}{\mathrm{d}s} = j_\nu - \chi_\nu I_\nu = 0$$

which leads to

$$\left.\begin{aligned} j_\nu &= \chi_\nu I_\nu \\ I_\nu &= B_\nu \end{aligned}\right\} \quad \text{(Kirchhoff's law)} \tag{11}$$

where $B_\nu(T)$ is the black body law,

$$\Rightarrow S_\nu = B_\nu = \frac{2h\nu^3}{c^2}\frac{1}{\exp(h\nu/kT)-1}\text{(Planck).}$$

Note that TE is strictly valid only for isotropic and isothermal medium ! This is rarely the case in astronomy.

Local Thermodynamic Equilibrium

In this case, TE is assumed locally. This means that one assumes that matter is in equilibrium with the local kinetic temperature, but the radiation may deviate (slightly) from this temperature and may vary slowly in the medium.
$\Rightarrow$ Maxwell, Boltzmann and Saha distributions are still valid as it is in equilibrium with the kinetic temperature.

$$\Rightarrow I_\nu \neq B_\nu(T).$$

However, the emitted energy is still given by the Planck function : $S_\nu = B_\nu(T)$ but the radiation is no longer necessarily isotropic ($J_\nu \neq B_\nu(T)$) and energy transport is allowed ($F_\nu \neq 0$).

Local Thermodynamic Equilibrium

LTE is valid when collisions dominate the energy partitioning of matter more strictly than for radiation $\Leftrightarrow$ LTE is valid when the destruction probability $\epsilon = 1$ (all photons are converted into kinetic energy of the gas) AND/OR $J_\nu = B_\nu(T)$.

- stellar interiors : $F_\nu \neq 0$ so not TE but high density $\Rightarrow$ many collisions $\Rightarrow \epsilon \sim 1 \Rightarrow$ LTE !
- stellar atmospheres : often $\epsilon << 1 \Rightarrow$ LTE not valid

Statistical Equilibrium

In this case, Saha and Boltzmann are no longer expected to hold. One must resort to solve the rate equations for ALL populations !

One usually assumes a steady state (equilibrium), hence $\frac{\mathrm{d}n_i}{\mathrm{d}t} = 0$.

The rate equations then write :

$$\frac{\mathrm{d}n_i}{\mathrm{d}t} = \sum_{j\neq i} n_j P_{ji} - n_i \sum_{j\neq i} P_{ij}$$

with $P_{ij} = A_{ij} + B_{ij} J_{\nu_0} + C_{ij}$ and A_{ij}, B_{ij} and C_{ij} are the Einstein coefficients. This states that the number of transitions up and down are equal for every level of each element !

Statistical Equilibrium

However C_{ij}, the coefficient associated to collisions, requires an integration over the velocity distribution of electrons. One usually assumes a Maxwell distribution for this.

Also, the rate equations contain J_ν's that are obtained from the transfer equation $\mu\frac{\mathrm{d}I_\nu}{\mathrm{d}\tau_\nu} = -S_\nu + I_\nu$ which can be solved only if the source term $S_\nu = j_\nu/\chi_\nu$ is known but it depends on the number of absorbing and emitting species, i.e. on the populations !
$\Rightarrow$ the rate equations and the transfer equation must be solved together (any transition depends on all other transitions).

Non-local Thermodynamic Equilibrium

By definition, NLTE is everything that is not LTE. Often SE is assumed and hence the populations are not necessarily given the Saha-Boltzmann distributions.

Since $\epsilon < 1$ (the radiation field is not determined by local conditions), the source function will have a scattering term in addition to the thermal emission given by $B_\nu(T)$:

$$S_\nu = \epsilon_\nu \cdot B_\nu + (1 - \epsilon_\nu) \cdot J_\nu .$$

Non-local Thermodynamic Equilibrium

ϵ_ν reflects the probability that a photon will be destroyed (i.e. the photon energy will be transformed to thermal energy and hence local temperature).

$(1-\epsilon_\nu)$ on the other hand is the probability that the photon will be scattered (no energy will be transferred) and therefore contains non-local information. In this later case, the photon may have a characteristic temperature (energy) very different from the local temperature.

If there is a lot of scattering, ϵ_ν will be small and hence S_ν will be very different from $B_\nu(T)$.

Eddington-Barbier approximation

This approximation states that the emergent intensity (at the surface) is given by the source function one mean optical depth from the surface.

It writes

$$I_\nu(\tau_\nu = 0, \mu) \sim S_\nu(\tau_\nu = \mu).$$

It is a very important approximation as it is often valid.

Note that this approximation is exact if S_ν is linear in τ_ν, e.g. $S_\nu(\tau_\nu) = a_0 + a_1 \cdot \tau_\nu$.

Thermalization depth

In the presence of scattering, J_ν drops below the Planck function close to the stellar surface. One talks about the thermalization depth where $J_\nu \sim B_\nu$: for deeper layers, photons can not escape via a scattering sequence until it reaches the surface.

The thermalization depth is defined as

$$\Lambda_\nu = \frac{1}{\sqrt{\epsilon_\nu}}.$$

Thermalization depth

We can have three different cases :

- Effectively optically thick : $\tau_\nu > \Lambda_\nu$

  $J_\nu \sim B_\nu$ and LTE is valid
- Optically thick : $1 < \tau_\nu < \Lambda_\nu$

  Photons can escape through scattering before being destroyed $\Rightarrow J_\nu \neq B_\nu$

  BUT the radiation field feels the presence of the surface
- Optically thin : $\tau_\nu < 1$

  Photons may escape immediately, the Eddigton-Barbier depth ($\tau_\nu = \mu$) characterizes the photon's last interaction NOT the location where they were created (typically Λ_ν).

Hints on constructing a model atmosphere

Constructing a model atmosphere is a complex process and so far we have discussed only the radiative transfer equation. A full model atmosphere relies on more assumptions :

1. Geometry
   - plane-parallel
   - spherical

→ only one spatial coordinate (z or r)

→ homogeneous structure assumed

2. Time-independent ⇔ steady-state
   - static or steady flow (stellar winds)
   - LTE or SE

Hints on constructing a model atmosphere

3 Momentum balance
- hydrostatic equilibrium (no motion)
- steady flow

4 Energy balance
- radiative equilibrium (no convection)
- flux constant (with convection)

5 Ideal gas
- relation between the pressure of the gas, the density and temperature

Hints on constructing a model atmosphere

6 Distribution of velocities
- Maxwell distribution valid

7 Magnetic fields (yes/no), waves, rotation etc.

Hints on constructing a model atmosphere

Momentum balance :

Hydrostatic equilibrium means

$$\frac{\mathrm{d}P_{total}}{\mathrm{d}\tau_0} = \frac{g}{\kappa_0}$$

where $P_{total} = P_{rad} + P_{gas}(+P_{magn} + P_{rot} + P_{turb})$ and κ_0 and τ_0 are the standard opacity and optical depth (Rosseland mean extinction or at 500nm).

Note that the radiation pressure is directed opposite to gravity and therefore partly compensates it. The effective gravity (felt by the gas) can be much reduced in hot stars and supergiants as $\frac{\mathrm{d}P_{gas}}{\mathrm{d}\tau_0} = \frac{g - g_{rad}}{\kappa_0} = \frac{g_{eff}}{\kappa_0}$ and $g_{rad} = \frac{1}{c}\int_0^\infty \kappa_\nu F_\nu \mathrm{d}\nu$.

Energy balance :

If one assumes time independence the transported energy is constant in both time and depth (flux constancy)

$$F_{total} = F_{rad} + F_{conv}(+F_{mech}) \qquad \text{and } F_{rad} = \int_0^\infty F_\nu \mathrm{d}\nu$$

$$\Rightarrow \nabla F_{total} = \frac{\mathrm{d}F_{total}}{\mathrm{d}z} = \frac{\mathrm{d}F_{total}}{\mathrm{d}\tau_0} = 0$$

Note, this approximation is valid only in plane-parallel geometry.

Hints on constructing a model atmosphere

Two cases :

- the entire energy flux is carried out by radiation : RADIATIVE EQUILIBRIUM

$\nabla F_{total} = \nabla F_{rad} = 0 \Leftrightarrow \int_0^\infty \chi_\nu (J_\nu - S_\nu) \mathrm{d}\nu = 0$ for each depth (Strömgren condition)

The total energy produced in the interior which needs to be transported is $F_{total} = \sigma T_{eff}^4 = \frac{L}{4\pi R^2}$.

Hints on constructing a model atmosphere

- the entire energy flux is carried by convection : CONVECTIVE EQUILIBRIUM

$F_{conv} = \rho \cdot C_P \cdot v \cdot \Delta T$ with C_P the specific heat at constant pressure.

Convection becomes more efficient at large densities. It is also favored by

- steep temperature gradient (high opacities)
- ionization
- important radiation pressure
- molecule formation

Construction of a model atmosphere :

1. guess $T(\tau_0)$ and $P_{gas}(\tau_0)$
2. calculate $P_e(\tau_0)$ iteratively from a $P_e - P_{gas}$ relation with the given abundances
3. calculate all necessary opacities $\kappa_i(\tau_0)$ from $T(\tau_0)$,$P_{gas}(\tau_0)$and$P_e(\tau_0)$ and from these opacities a standard opacity κ_0
4. calculate $P_{rad}(\tau_0)$ from $I_\nu(\tau_0)$ solving the transfer equation
5. calculate the new pressure structure by integrating the hydrostatic equilibrium equation using $P_{total}(\tau_0) = P_{gas}(\tau_0) + P_{rad}(\tau_0)$, κ_0 and the surface gravity

Hints on constructing a model atmosphere

Construction of a model atmosphere :

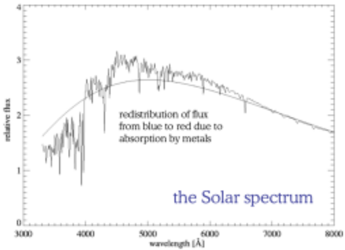
6. solve the transfer equation for all frequencies to obtain $I_\nu(\tau_0), F_\nu(\tau_0), F_{rad}(\tau_0)$ and $P_{rad}(\tau_0)$
7. calculate the convective flux $F_{conv}(\tau_0)$
8. if the total flux $F_{total}(\tau_0) = F_{rad}(\tau_0) + F_{conv}(\tau_0)$ differs from σT^4_{eff}, correct $T(\tau_0)$ and iterate from step 2 until convergence (step 4 can be omitted and P_{rad} from step 6 can be used instead in step 5).

Spectral lines

Effect on the temperature gradient :

Temperature stratification is one of the key ingredient of building a stellar atmosphere model.

However, not all processes are continuum processes and bound-bound transitions of atoms or molecules in the photosphere will affect the temperature.



Spectral lines

Why do these spectral lines affect the temperature stratification ?

Strong lines mean additional opacity $\Rightarrow$ less flux is transported at the wavelengths of spectral lines.

The total flux σT_{eff}^4 is conserved $\Rightarrow$ more flux must be carried out by the continuum wavelengths $\Rightarrow$ the temperature must increase at those optical depth τ_0 where this flux originates (if fact $\tau_0 \geq 2/3$).

More difficult to push through the radiation $\Rightarrow$ the temperature gradient must also be larger (backwarming process).

Spectral lines

At the surface, spectral lines can either cool or heat depending on the sign of $\chi_\nu(S_\nu - I_\nu)$ with positive values for cooling and negative values for heating.

Strong lines have $\mathrm{d}\tau_\nu^{total} = \left(1 + \kappa_\nu^{line}/\kappa_\nu^{continuum}\right)\mathrm{d}\tau_\nu^{cont}$ which means that for LTE, they usually cool the surface. The escaping line photons take thermal energy away from their place of creation.

Since flux constancy is required, fewer continuum photons must be created $\Rightarrow$ the temperature must decrease at the surface.

Effect of backwarming : a sketch (Zwang 1993) :

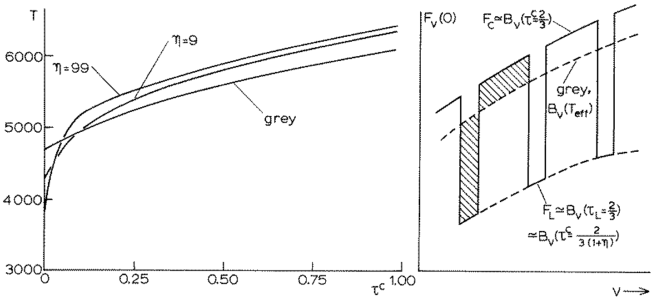


Figure 7.6: LTE backwarming and surface cooling using a schematic "picket fence" model in which the flux-blocking fraction f consists of equidistantly-spaced rectangular lines, all with the same extinction and resembling pickets in a fence. They occupy 20% of the spectral bandwidth; their strength is given by $\eta = \kappa^l_\nu/\kappa^c_\nu$. The lines are formed in LTE, as is the continuum. The blocking causes backwarming and a higher flux for the continuum between the lines than in the grey case. At the surface the lines cause appreciable cooling because they are in LTE; their photon losses deplete the thermal pool locally. The righthand graph shows the spectrum. The Eddington-Barbier depths for the emergent flux are $\tau^c_\nu = 2/3$ for the continuum windows and $\tau^c_\nu = (2/3)(1/1+\eta_\nu)$ for the lines, with total optical depth $\mathrm{d}\tau_\mathrm{L} = \mathrm{d}\tau^c_\nu + \mathrm{d}\tau^l_\nu = (1+\eta_\nu)\,\mathrm{d}\tau^c_\nu$. From Zwaan (1993).

Spectral lines

Effect of metallicity on the temperature structure of stellar atmospheres. MARCS models Gustafsson et al. 2008

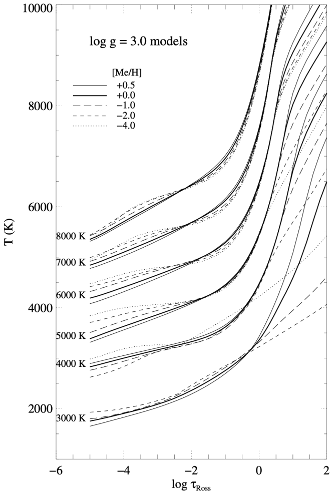


Fig. 2. The temperature structures for a set of model atmospheres with different $T_{\rm eff}$, $\log g = 3$ and different metallicities.

Spectral lines

If the line predominantly scatter ($\epsilon_\nu^{line} \ll 1$) then the temperature is little affected close to the surface : $S_\nu = \epsilon_\nu B_\nu + (1-\epsilon_\nu) J_\nu \sim J_\nu$.

The reason is that escaping photons have taken thermal energy from deeper layers, not from the surface. The amount of backwarming remains the same.

Line absorption profiles :

Bound-bound transitions give rise to spectral lines with a typical energy difference $\Delta E \sim$ a few eV in the visible range. Lines are not infinitely narrow but have a width caused by :

- natural broadening or radiation broadening
- pressure or collisional broadening
- Doppler broadening by thermal motion
- Doppler broadening by non-thermal motion (convection)
- rotational broadening
- ...

The width of the lines make them able to absorb photons with slightly different wavelength λ than the central wavelength λ_0.

Natural broadening/radiation broadening

Any atomic level in a line transition has a finite lifetime τ (except the ground state) and due to Heisenberg uncertainty principle has an uncertainty in energy

$$\left.\begin{array}{r}\tau \cdot \Delta E \geq \hbar \\ E = \hbar\omega = h\nu\end{array}\right\} \Rightarrow \Delta\omega = \gamma^{rad} = \gamma_u^{rad} + \gamma_l^{rad} = \frac{1}{\tau_u} + \frac{1}{\tau_l}$$

Spectral lines

The broadening arises due to the interaction of an electromagnetic wave (the photon) with an oscillating dipole (absorbing atom) ⇒ damped harmonic oscillator. The resulting damping profile is described by a Lorentz profile :

$$\alpha_\lambda^{line} = \alpha_0 \phi(\lambda)$$

with $\alpha_0 = \frac{\pi e^2}{4\pi\epsilon_0 m_e c^2}\lambda_0^2 f$, $\phi(\lambda) = \frac{1}{\pi}\frac{\Gamma}{(\lambda-\lambda_0)^2+\Gamma^2}$ and $\Gamma = \gamma^{rad}\lambda_0^2/c$.

γ^{rad} is the damping constant and f is the oscillator strength (~ 1 for strong allowed lines, 10^{-10} for forbidden lines).

Note that $\phi(\lambda)$ is normalized (e.g. $\int \phi(\lambda) \mathrm{d}\lambda = 1$).

Pressure/collisional broadening

The absorbing/emitting atom A may be disturbed by a passing particle B (atom/ion/electron) which modifies the energy levels and transitions by electromagnetic interaction with the atom.

The energy levels of atom A are altered due the collision according to $\Delta E \propto \frac{1}{r^n} \Rightarrow \Delta\lambda$ where r is the separation between atom A and the perturber B.

n can take different value, depending on the type of interaction

Spectral lines

$n = 2$: linear Stark broadening

A = one electron system (HI, HeII)
B = charged particle (electron or ion)
This causes the very wide broadening of HI lines in hot stars

$n = 3$: resonance broadening

A=B (i.e. collisions HI HI)
Important for solar Balmer lines of HI

$n = 4$: quadratic Stark broadening → Lorentz profile γ_4

A = many electron system
B = charged particle (electron or ion)
Affects most lines other than HI in hot stars

$n = 6$: Van der Waals broadening → Lorentz profile γ_6

A = many electron system
B = neutral particle (usually H or He)
The most important broadening for almost all lines in cool stars as H and He are neutral

Doppler broadening

Due to thermal motion the absorbing/emitting atom experiences Doppler shifts $\frac{\Delta\lambda}{\lambda} = -\frac{v_r}{c}$.

The number of atoms with a given velocity is (normally) given by the Maxwell velocity distribution

$$\frac{n(v_r)}{N} \mathrm{d}v_r = \frac{1}{\sqrt{\pi} V_D} \exp\left(-\frac{v_r^2}{V_D^2}\right) \mathrm{d}v_r,$$

with V_D the Doppler velocity ($V_D = \sqrt{\frac{2kT}{m_A}}$, m_A the mass of the atom).

The absorption profile must therefore be convolved with the velocity distribution

$$\alpha_\nu = \int_{-\infty}^{+\infty} \alpha' \left(\nu - v_r \frac{\nu}{c}\right) \frac{n(v_r)}{N} \mathrm{d}v_r.$$

There is also additional motion due to turbulence (non-thermal) arising from the convective motion which causes also a Doppler shift. In this case, we introduce the microturbulence parameter ξ_{turb} and redefine the Doppler velocity as $V_D = \sqrt{\frac{2kT}{m_A} + \xi_{turb}^2}$. This is an had-oc parameter and a proper treatment requires a full 3D treatment of the model atmosphere.

Combining broadening mechanism

The total broadening of a line is obtained by convolving all different mechanisms :

$$\alpha_\nu^{total} = \alpha_\nu^{rad} * \alpha_\nu^{coll} * \alpha_\nu^{thermal}.$$

α_ν^{rad} is a Lorentz profile and so is α_ν^{coll} if it is due to quadratic Stark or Van der Waals broadening.

$\alpha_\nu^{thermal}$ on the other hand is a Gaussian profile.

Spectral lines

Combining a Lorentz with Gaussian gives a Voigt profile :

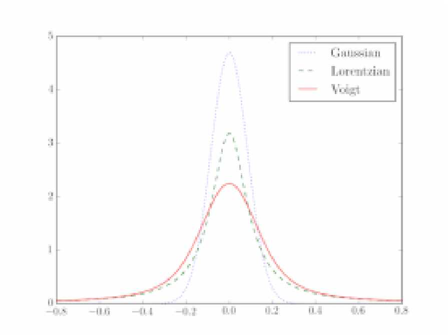
$$\alpha_\nu = \frac{\pi e^2}{m_e c} f V(u,a)$$

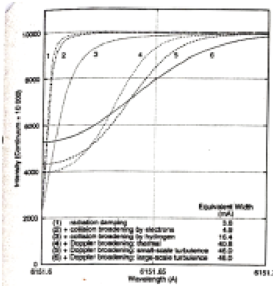
where

$$V(u,a) = \frac{a}{\pi^{3/2}\Delta\lambda_D}\int_0^{+\infty}\frac{e^{-t^2}}{(u-t)^2+a^2}\mathrm{d}t,$$

with $u = \Delta\lambda/\Delta\lambda_D$, $a = \Gamma/\Delta\lambda_D$ and $\Delta\lambda_D = \frac{\lambda}{c}V_D$.

Spectral lines





Holweger 1996

Line profile contribution

Spectra lines mean additional opacity compared to the continuous opacity at the line wavelength. If one calls l_ν the line opacity, we have

$$l_\nu = N_{lji}\alpha_0\Phi_\nu/\rho \qquad ([l_\nu] = \mathrm{cm}^2/\mathrm{g})$$

with N_{lji} the number of atoms in a lower level l, ionization stage j for element i, Φ_ν the absorption profile and ρ the density. The total opacity is given by

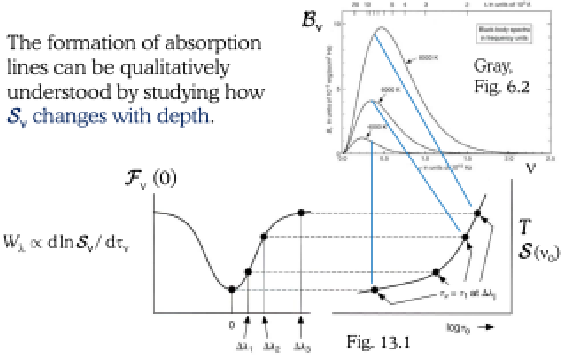
$$\kappa_\nu^{total} = \kappa_\nu^{cont} + l_\nu,$$

which translates in the total optical depth

$$\tau_\nu^{total} = \tau_\nu^{cont} + \tau_\nu^{line} = -\int \kappa_\nu^{cont}\rho \mathrm{d}z - \int l_\nu \rho \mathrm{d}z.$$

Spectral lines

This change in depth, the fact that a spectral line has a finite width and its relation to the source function is shown below

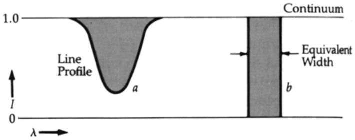


(see also Gray, Fig 13.3, p.279)

Equivalent width

We define the equivalent width as the width of a normalized continuum (normalized to 1) that has the same area as the line, e.g.

$$W = \int \frac{F_{cont} - F_\nu}{F_\nu} \mathrm{d}\lambda \qquad ([W] = \mathrm{pm}).$$



For weak lines this quantity is directly proportional to the number of atoms of element i : $W \propto \frac{f\, N_{lji}}{\kappa_\nu^{cont}}$.

Behavior of spectral lines on fundamental parameters

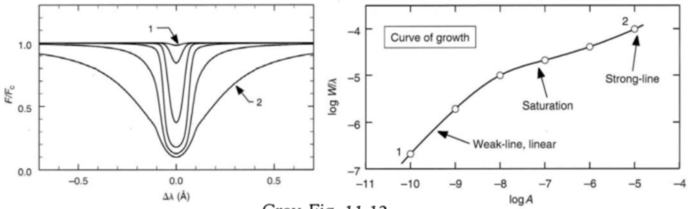
Abundances

There are three cases depending on the line strength

- Weak lines : $W \propto A_i^1$ where A_i is the abundance of the element $\frac{N_i}{N_H}$ (in fact $W \propto N_{l\,j\,i}$)
- Strong lines : $W \propto A_i^0$, there is no dependence on abundance because of the saturation of the line
- Very strong lines : $W \propto A_i^{1/2}$, absorption in the wings of the line.

Behavior of spectral lines on fundamental parameters

This is summarized in the curve-of-growth



Gray, Fig. 11.13

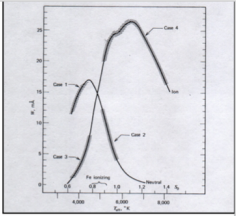
Note that for abundance determination in stars, weak lines are preferred not only because of the linear dependence but also because these lines are probably less affected by departures from LTE as they originates further in the star where the density is higher.

Behavior of spectral lines on fundamental parameters

Temperature

Essentially all lines have a certain sensitivity to temperature via Boltzmann and the continuous opacity is increased (reducing the layer of line formation). The exact details depend on the excitation potential and ionization potential but 4 main cases can be identified.

The Effect of Temperature on Weak Lines



1. neutral line, mostly neutral species
2. neutral line, mostly ionized species
3. ionic line, mostly neutral species
4. ionic line, mostly ionized element

Behavior of spectral lines on fundamental parameters

Gravity

When the gas pressure increases, the electron pressure normally also increases (for the Sun $P_e \propto \sqrt{P_{gas}}$) which has two effects :

- less ionization $\Rightarrow l_\nu/\kappa_\nu^{cont}$ changes
- more pressure damping

Behavior of spectral lines on fundamental parameters

Three cases can be identified :

1. weak neutral lines formed by an ion or atom where most of the element is in the next higher ionization stage are pressure insensitive :

$$l_\nu/\kappa_\nu^{cont} \propto (N_i \cdot P_e)/(P_e \cdot N_H) \propto N_i/N_H$$

2. weak lines formed by an ion or atom where most of the element is in the same ionization stage are pressure sensitive :

$$l_\nu/\kappa_\nu^{cont} \propto N_i/(P_e \cdot N_H) \propto (A_i \cdot N_H)/(N_H \cdot P_e) \propto 1/g^{1/3}$$

so that when

$$g \searrow \Rightarrow P_{gas} \searrow \Rightarrow P_e \searrow \Rightarrow \kappa_\nu^{cont} \searrow \Rightarrow l_\nu/\kappa_\nu^{cont} \nearrow \Rightarrow W \searrow$$

Behavior of spectral lines on fundamental parameters

Three cases can be identified :

3 weak lines formed by an ion or atom where most of the element is in the next lower ionization stage are VERY pressure sensitive :

$$l_\nu / \kappa_\nu^{cont} \propto (N_i / P_e)/(N_H / P_e) \propto 1/P_e^2 \propto 1/g^{2/3}$$

$$g \searrow \Rightarrow W \nearrow$$

Therefore we can compare neutral and ionized lines of the same element in a star to determine the luminosity class of a star $(\log g)$.

Stellar parameter determination

Determination of stellar parameters

Temperature determinations

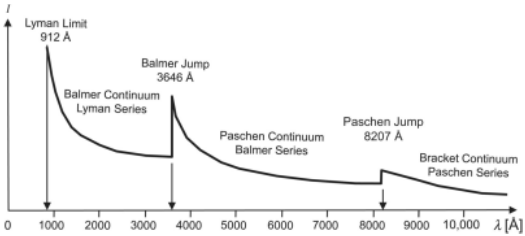
1. Absolute flux method :

$$T_{eff} = \left(\frac{1}{\sigma} \int_0^\infty F_\nu^{earth} \mathrm{d}\nu \right)^{1/4} \Big/ (R_*/d)^{1/2}$$

to use this method, we need to know the radius of the star and its distance !
Also F_ν^{earth} is difficult to measure because of the earth's atmosphere.

2 Paschen and Balmer continuum :

The slopes of the Balmer and Paschen continuum are temperature dependent (ionization of H from n=3 or n=2). If no interstellar reddening is present it is relatively independent of the spectra lines and also unlikely affected by complications of non-LTE.



Determination of stellar parameters

Higher $T_{eff} \Rightarrow$ steeper F_λ since $F_\lambda \sim B_\lambda(T_{eff})$.

T_{eff} measurements for

- $T_{eff} < 6000K$: H^- absorption + line opacity prevent the use of the method
- $6000K \leq T_{eff} \leq 10000K$: Paschen (from spectral type G0 to A0)
- $10000K < T_{eff}$: Balmer continuum

Determination of stellar parameters

For main sequence star with $T_{eff} < 10000K$ we have

$$\frac{\partial \log(F_{4000}/F_{7000})}{\partial \log T_{eff}} \sim 2.3.$$

Taken literally, it means that if we can measure the continuum slope between $\lambda 4000$ and $\lambda 7000$ to 2.3%, then the temperature is established to $\pm 1\%$.

Determination of stellar parameters

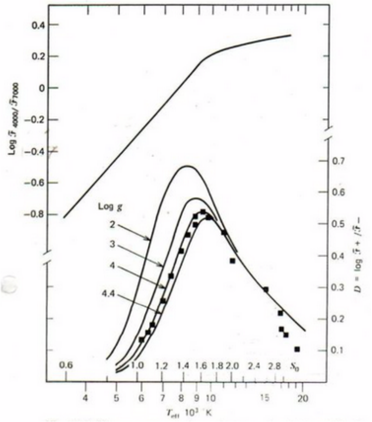
3 Balmer jump :

For hot star the temperature dependence of the Balmer jump (or discontinuity) is a good indication of temperature.

$$D_B = 2.5 \log(F_{3647+}/F_{3647-})$$

It measures the change in opacity due to the onset of H_{bf} absorption from the $n = 2$ level. At low wavelength, because of the increased opacity, the radiation originates from higher level in the photosphere where the temperature is cooler ⇒ less flux. At higher wavelength, we see deeper in the photosphere ⇒ higher temperature, more flux.

Determination of stellar parameters



Determination of stellar parameters

For cooler stars ($T_{eff} < 10000K$), there is a gravity sensitivity and the slope changes with respect to this quantity from 2.3 to 2.8. An error of 5% on the flux ratio induces an error of ~2% on T_{eff} is there is no error on gravity.

For hot stars, the gravity sensitivity disappears and the slope is smaller (monotonic decrease), hence larger errors on the temperature determination.

In practice, it is not easy to measure the Balmer jump because the hydrogen lines blend together at the series limit causing distortion in the Paschen continuum.

One can however fit the Paschen continuum towards longer wavelength and the Balmer continuum below the jump.

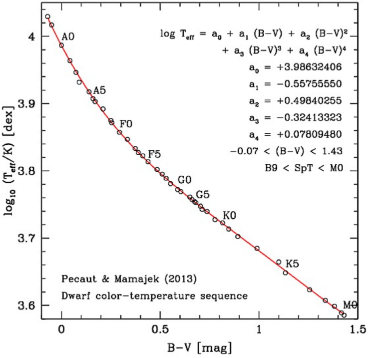
Note that the depth of formation of F_+ and F_- is sufficently different that the size of the Balmer jump depends on the temperature gradient in the photosphere $\Rightarrow$ theoretical models can differ in their relation between the Paschen-continuum slope and the Balmer jump. This can lead to ambiguity in the interpretation of observations.

For very cool stars, chemical composition can also play a role as the number of electron donors can alter the opacity.

Determination of stellar parameters

4 Photometry :

Broad-band colors (B-V, V-R,V-K) measures the flux ratio in different wavelength intervals. It is therefore temperature dependent.



Determination of stellar parameters

Problems :

- need to be calibrated against stars for which T_{eff} is already known
- depends on the star luminosity class
- depends on abundances

Accuracy of this method ~2% but can be of the order of 50-100K with the newest observations (APASS, SDSS...).

5 Infrared flux method :

This is an extreme method where we compare the total flux with an infrared monochromatic flux

$$Q = \frac{\int F_\nu^{earth} \mathrm{d}\nu}{F_\nu^{earth}(\nu_1 = IR)} = \frac{\sigma T_{eff}^4}{f_\nu^{model}(\nu_1 = IR)}$$

The first equality is the observed quantity while the last part of the equation are model predictions.
The infrared is chosen because of the Rayleigh-Jeans regime where $F_\nu \propto T_{eff} \Rightarrow Q \propto T_{eff}^3$.

6 Hydrogen lines :

The strength of the Balmer lines (H_α,H_β, H_γ...) are good temperature diagnostics for $T_{eff} \leq 8000K$ due to the necessary excitation to n=2 :

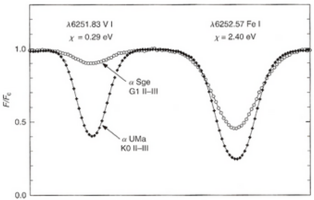
$$T_{eff} \nearrow \Rightarrow \text{more excitation} \Rightarrow W \nearrow .$$

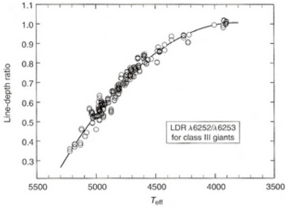
For temperatures above this limits, the lines are also gravity sensitive.

Problem : this lines are very broad, hence W is difficult to measure. It is better to fit the line profile.

7 Line ratios :

The temperature can be estimated by comparing line ratios (line depth or W) as lines from the same element should be also be pressure insensitive (but not quite true)





8 Flux distribution fitting :

the goal here is to fit the flux from model atmospheres to the observed spectra where the stellar flux peaks (UV for hot stars, optical to IR for cool stars)

Problems : observations are hard, molecular bands in the IR for cool stars etc.

Determination of stellar parameters

Gravity determination

Pressure or $\log g$ is hard to measure in stars. Most diagnostics are also temperature dependent and hence require an accurate measurement of T_{eff}.

Here I will list a few methods with their problems for single stars.

1 Continuum features :

Only the Balmer jump is sufficiently pressure sensitive but as we saw before, it is also temperature dependent.

Accuracy ~50% but if T_{eff} is wrong $\Rightarrow \log g$ very wrong.

Determination of stellar parameters

2 Hydrogen lines :

The wings of hydrogen lines are pressure broadened but they are also temperature sensitive

Needs accurate T_{eff} as well as the abundances of metals as it depends also on P_e

3 Strong lines (besides Hydrogen) :

Any strong line with pressure broadened wings can be used (typically Ca or Na).

As for H, they are temperature and abundances sensitive

Can not be applied to giants and supergiants where natural broadening is more important.

Determination of stellar parameters

4 Weak lines :

The ionization balance of elements is sensitive to gravity. We can compare the line ratios of element in two different ionization stages.

As for the other methods, it requires T_{eff} to be accurately known as well as the detailed abundances.

Abundances :

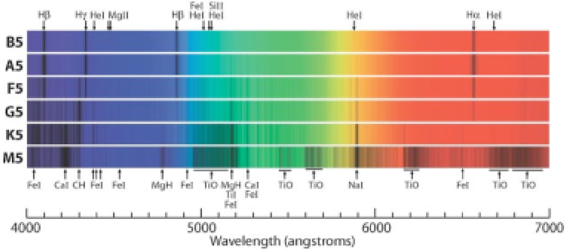
Beyond the scope of this lecture.

Many methods exist, curve of growth, direct profile fitting etc. Which method to use depends usually on the observed spectra (low/high resolution, signal to noise ratio, fluxed spectra or not...)

Spectral classification

Spectral classification of stars

The historical classification of stars is based on the strength of the spectral features and at the time energy level structure of atoms was unknown.



Spectral classification of stars

In this Morgan Keenan (MK) classification scheme, star are ordered in a sequence including 7 categories : O, B, A, F, G, K, M. This sequence was solely based on the progression of line patterns.

Later, Annie Cannon added subclasses to this sequence, dividing each category in 10 sub-categories numbered from 0 to 9.

Using this ordering, the sequence is actually describing the temperature of stars (O being the hotter, M the cooler).

Spectral classification of stars

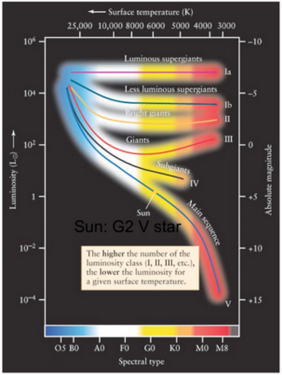
We have seen in previous chapters that star evolve during their lifetime and the HR diagram (T-L diagram) that for a given temperature, stars can have different luminosity according to their evolutionary stage.

Hence, a roman number has been added to the classification to take this into account, the luminosity class.

Spectral classification of stars

The Morgan-Keenan Luminosity Class

L. Class	Star
Ia	Luminous supergiant
Ib	supergiant
II	bright giant
III	giant
IV	subgiant
V	main sequence

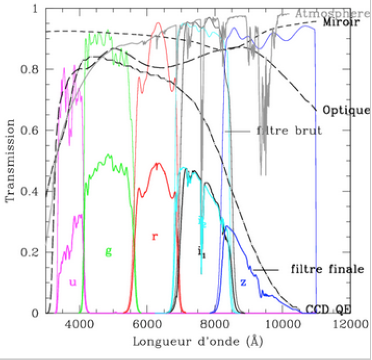


Photometry of stars

Instruments do not have a uniform sensitivity at all wavelength. The ratio of the incoming illuminance to the measured signal is named instrumental response. It depends on many parameters such as

- mirrors reflection coefficients
- transmission coefficients of the lenses
- sensitivity of the detectors
  - dependency on λ

We also use filters that limit the passband.



Hence, to compare measurements on different telescopes, best is to adopt standard filters and make sure that the λ dependency of the instrument's response is dominated by the filter response.

Photometry of stars

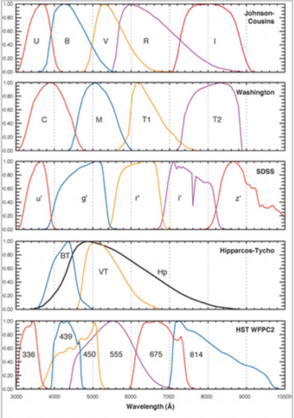
Filters are characterized by their transmission $T(\lambda)$. On a synthetic manner by their central wavelength and width.

We define :

- central wavelength $\lambda_0 = \frac{\int \lambda.T(\lambda)\mathrm{d}\lambda}{\int T(\lambda)\mathrm{d}\lambda}$
- Effective wavelength $\lambda_{eff} = \frac{\int \lambda.T(\lambda).S(\lambda)\mathrm{d}\lambda}{\int T(\lambda).S(\lambda)\mathrm{d}\lambda}$ with $S(\lambda)$ the observed spectra

We also note FWHM the full width at half maximum.

Photometry of stars



Example of classical filters

Photometry of stars

One of the historically most used photometric system is the Johnson-Cousin system (broad-band) described below in the table. Nowadays, new system exists and are more widely used (u,g,r,i,z)

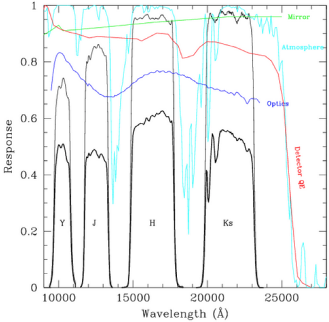
Photométrie standard

Dénomination	λ_0 (μm)	$\Delta\lambda_0$ (μm)	e_0(W m^{-2} μm^{-1})	e_0' (Jy)	
U	0.36	0.068	4.35 E-8	1 880	ultraviolet
B	0.44	0.098	7.20 E-8	4 650	bleu
V	0.55	0.089	3.92 E-8	3 950	visible
R	0.70	0.22	1.76 E-8	2 870	rouge
I	0.90	0.24	8.3 E-9	2 240	infrarouge
J	1.25	0.30	3.4 E-9	1 770	infrarouge
H	1.65	0.35	7 E-10	636	infrarouge
K	2.20	0.40	3.9 E-10	629	infrarouge
L	3.40	0.55	8.1 E-11	312	infrarouge
M	5.0	0.3	2.2 E-11	183	infrarouge
N	10.2	5	1.23 E-12	43	infrarouge
Q	21.0	8	6.8 E-14	10	infrarouge

1 Jy = 10^{-26} Wm^{-2} Hz^{-1}.

Why do we use filters ?

Extinction by the earth atmosphere is actually a problem for ground observations. We use filters to avoid regions of high atmospheric extinction as it varies with time and location. Atmospheric extinction is mostly due to H2O absorption.



Apparent magnitude through a filter

Given ξ_λ, the monochromatic illuminance, with $\int_\lambda \xi_\lambda \,\mathrm{d}\lambda = \xi$, the apparent magnitude through a filter B is given by

$$m_B = -2.5 \log_{10} \frac{\int_\lambda T_B(\lambda).\xi_\lambda \,\mathrm{d}\lambda}{\int_\lambda T(\lambda)\,\mathrm{d}\lambda} + c_B$$

which we can also write

$$m_B = -2.5 \log_{10} \frac{\xi_B}{e_0(B)}.$$

$e_0(B)$ in $W.m^{-2}.\mu m^{-1}$, the m comes from the collecting surface and μm from the wavelength.

Apparent magnitude through a filter

If one uses frequency (as in radio astronomy), this rewrites

$$
\begin{aligned}
m_B &= -2.5\log_{10}\frac{\int_\nu T_B(\nu).\xi_\nu\,\mathrm{d}\nu}{\int_\nu T_B(\nu)\,\mathrm{d}\nu} + c'_B \\
&= -2.5\log_{10}\frac{\xi'_B}{e'_0(B)}
\end{aligned}
.
$$

Note that c_B and c'_B are different and depend on the chosen unit.

Also, many photometric system coexist that adopt identical filters but different constants.

Apparent magnitude through a filter

- **Classical system (historic) : Vega**

The constants (c, c', e_0, e_0') are chosen such that Vega (α Lyrae) has (quasi-) zero magnitudes in all filters. Historically, $m = 0$ but today we use $m = 0.03$ instead.

Hence, $m_B - 0.03 = -2.5 \log_{10} \frac{\xi_B}{e_0(B)}$, with $e_0 = \xi_B(Vega)$.

Apparent magnitude through a filter

Practical implication :

- at the telescope, one must observe the scientific target but also Vega !
- in reality we observed sets of photometric standard stars whose magnitudes are well known for the chosen system
- a star that transmits as much energy through the filter as Vega will have the same magnitude (0)
  - if the star is brighter $m < 0$
  - if the star is fainter $m > 0$

Apparent magnitude through a filter

- **Modern systems : suited to CCDs and photon counting instruments**

In the classical system (photographic plates), two sources are given the same magnitude if we measure as much energy, for a fixed exposure time, through the filter.

With CCDs, stars will have the same magnitude if they produce the same number of photons on the detector.

Apparent magnitude through a filter

To an illuminance $\xi_\lambda([erg.s^{-1}.cm^{-2}.\mathring{A}^{-1}])$ corresponds a monochromatic photon flux $\frac{\xi_\lambda}{h\nu} = \frac{\xi_\lambda.\lambda}{hc}([photon.s^{-1}.cm^{-2}.\mathring{A}^{-1}])$.

It induces a modification of the formula for magnitudes. For B we have $m_B = -2.5\log_{10}\frac{\int_\lambda \xi_\lambda.\lambda T_B(\lambda)\,\mathrm{d}\lambda}{\int_\lambda \lambda T_B(\lambda)\,\mathrm{d}\lambda} + cste$.

The quantity $\frac{\int_\lambda \xi_\lambda.\lambda T_B(\lambda)\,\mathrm{d}\lambda}{\int_\lambda \lambda T_B(\lambda)\,\mathrm{d}\lambda}$ is proportional to the photon flux via λ at the numerator.

Apparent magnitude through a filter

Note :

- λ is NOT fundamentally required, it is a convention.
- This ratio has the same unit as ξ_λ (energy by... instead of photon by...).
- It is a mean of ξ_λ weighted by $\lambda.T(\lambda)$ not to confuse with the classical weighting by $T(\lambda)$.

Apparent magnitude through a filter

Modern photometric systems :

- VegaMag : classical analog for CCD ; $mx(Vega) = 0$
- STMAG : suited to work with wavelength
  STMAG(filter)= $-2.5\log10(\xi_{filter}) - 21.10$
  same constant for all filters
- ABMAG : suited to work with frequencies
  ABMAG(filter)= $-2.5\log10(\xi'_{filter}) - 48.60$

Constants were chosen such that the magnitude of an object in the V band are close from one system to another.

Color indices

It is the ratio of the flux emitted by an object at two different wavelength (more specifically illuminance received at earth). It is a magnitude difference, for example : $B - V = m_B - m_V$

In Johnson-Cousin photometry or VegaMag photometry, all color indices of Vega equal to zero, Vega : A0V star ($T_{eff} \sim 10000K$).

When computing a color index, the convention is to put the bluest magnitude on the left. For example : B-V and never V-B, K-J, I-R...

Color indices

If a color indice in VegaMag is

< 0 : star bluer than Vega

> 0 : star redder than Vega.

Note :

Vega is a blue star.

For STMAG or ABMAG photometry, color indices of Vega are not zero !

Bolometric correction

For a filter X the bolometric correction is defined as

$$BC_X = m_{bol} - m_X.$$

It measures the ratio between the total flux of the object (bolometric) and the flux in the band X.

Note :

- heavy dependency on the temperature and evolutionary stage of the star.
- depends also on the photometric system.

Absolute magnitude

It is the magnitude of the object if its distance is 10pc. Hence the relation between the apparent and absolute magnitude (here given for the V band)

$$M_v = m_v - 5\log_{10} d + 5 + A_v$$

where d is the distance to the object in pc and A_v is the extinction in the V band in mag.

The quantity $m - M$ is called the distance modulus.

Absolute magnitude

Note :

- The observational equivalent of the theoretical HR-diagram (T_{eff}vs$\mathscr{L}$ is the color-magnitude diagram (for example V vs. $B-V$).
- Colors are proxys of temperature (the bluer the object, the hotter its temperature) while magnitude is minus 2.5 the logarithm of a flux, so proportional to minus a luminosity.

MK standards (Allen's astrophysical quantities)

Table 15.7. *Calibration of MK spectral types.*

Sp	M(V)	B − V	U − B	V − R	R − I	T_{eff}	BC
MAIN SEQUENCE, V							
O5	−5.7	−0.33	−1.19	−0.15	−0.32	42 000	−4.40
O9	−4.5	−0.31	−1.12	−0.15	−0.32	34 000	−3.33
B0	−4.0	−0.30	−1.08	−0.13	−0.29	30 000	−3.16
B2	−2.45	−0.24	−0.84	−0.10	−0.22	20 900	−2.35
B5	−1.2	−0.17	−0.58	−0.06	−0.16	15 200	−1.46
B8	−0.25	−0.11	−0.34	−0.02	−0.10	11 400	−0.80
A0	+0.65	−0.02	−0.02	0.02	−0.02	9 790	−0.30
A2	+1.3	+0.05	+0.05	0.08	0.01	9 000	−0.20
A5	+1.95	+0.15	+0.10	0.16	0.06	8 180	−0.15
F0	+2.7	+0.30	+0.03	0.30	0.17	7 300	−0.09
F2	+3.6	+0.35	0.00	0.35	0.20	7 000	−0.11
F5	+3.5	+0.44	−0.02	0.40	0.24	6 650	−0.14
F8	+4.0	+0.52	+0.02	0.47	0.29	6 250	−0.16
G0	+4.4	+0.58	+0.06	0.50	0.31	5 940	−0.18
G2	+4.7	+0.63	+0.12	0.53	0.33	5 790	−0.20
G5	+5.1	+0.68	+0.20	0.54	0.35	5 560	−0.21
G8	+5.5	+0.74	+0.30	0.58	0.38	5 310	−0.40
K0	+5.9	+0.81	+0.45	0.64	0.42	5 150	−0.31
K2	+6.4	+0.91	+0.64	0.74	0.48	4 830	−0.42
K5	+7.35	+1.15	+1.08	0.99	0.63	4 410	−0.72
M0	+8.8	+1.40	+1.22	1.28	0.91	3 840	−1.38
M2	+9.9	+1.49	+1.18	1.50	1.19	3 520	−1.89
M5	+12.3	+1.64	+1.24	1.80	1.67	3 170	−2.73
GIANTS, III							
G5	+0.9	+0.86	+0.56	0.69	0.48	5 050	−0.34
G8	+0.8	+0.94	+0.70	0.70	0.48	4 800	−0.42
K0	+0.7	+1.00	+0.84	0.77	0.53	4 660	−0.50
K2	+0.5	+1.16	+1.16	0.84	0.58	4 390	−0.61
K5	−0.2	+1.50	+1.81	1.20	0.90	4 050	−1.02
M0	−0.4	+1.56	+1.87	1.23	0.94	3 690	−1.25
M2	−0.6	+1.60	+1.89	1.34	1.10	3 540	−1.62
M5	−0.3	+1.63	+1.58	2.18	1.96	3 380	−2.48

Sp	M(V)	B − V	U − B	V − R	R − I	T_{eff}	BC
SUPERGIANTS, I							
O9	−6.5	−0.27	−1.13	−0.15	−0.32	32 000	−3.18
B2	−6.4	−0.17	−0.93	−0.05	−0.15	17 600	−1.58
B5	−6.2	−0.10	−0.72	0.02	−0.07	13 600	−0.95
B8	−6.2	−0.03	−0.55	0.02	0.00	11 100	−0.66
A0	−6.3	−0.01	−0.38	0.03	0.05	9 980	−0.41
A2	−6.5	+0.03	−0.25	0.07	0.07	9 380	−0.28
A5	−6.6	+0.09	−0.08	0.12	0.13	8 610	−0.13
F0	−6.6	+0.17	+0.15	0.21	0.20	7 460	−0.01
F2	−6.6	+0.23	+0.18	0.26	0.21	7 030	−0.00
F5	−6.6	+0.32	+0.27	0.35	0.23	6 370	−0.03
F8	−6.5	+0.56	+0.41	0.45	0.27	5 750	−0.09
G0	−6.4	+0.76	+0.52	0.51	0.33	5 370	−0.15
G2	−6.3	+0.87	+0.63	0.58	0.40	5 190	−0.21
G5	−6.2	+1.02	+0.83	0.67	0.44	4 930	−0.33
G8	−6.1	+1.14	+1.07	0.69	0.46	4 700	−0.42
K0	−6.0	+1.25	+1.17	0.76	0.48	4 550	−0.50
K2	−5.9	+1.36	+1.32	0.85	0.55	4 310	−0.61
K5	−5.8	+1.60	+1.80	1.20	0.90	3 990	−1.01
M0	−5.6	+1.67	+1.90	1.23	0.94	3 620	−1.29
M2	−5.6	+1.71	+1.95	1.34	1.10	3 370	−1.62
M5	−5.6	+1.80	+1.60:	2.18	1.96	2 880	−3.47

The End !

Next step examination : 1h quiz, no document allowed.