

# Perturbation growth within self-gravitating interstellar filaments

Jonathan Freundlich<sup>1,2</sup>, Chanda J. Jog<sup>3</sup>, and Françoise Combes<sup>1,4</sup>

<sup>1</sup>LERMA, Observatoire de Paris, PSL Research University, CNRS, Sorbonne Universités, UPMC Univ. Paris 06, F-75014, Paris, France; <sup>2</sup>IFCPAR/CEFIPRA Raman-Charpak fellow; <sup>3</sup>Indian Institute of Science, Bangalore, India; <sup>4</sup>Collège de France, PSL Research University, F-75005, Paris, France

**Abstract.** Observations show that the interstellar medium hosts complex networks of filamentary structures. The formation of turbulence-driven filaments and their subsequent gravitational fragmentation could represent an important step towards core and star formation. Gravitational instabilities could indeed lead these filaments to clump into a series of bead-like structures which would then turn into stars. To investigate the growth of such instabilities and the properties of the resulting substructures, we consider idealized self-gravitating filaments and derive the dispersion relation for perturbations within them. We assume no specific density distribution and use hydrodynamics to derive the linearized equations that govern the perturbations' growth. Assuming local perturbations leads to a dispersion relation analogous to the spherical Jeans case, but all modes are potentially unstable for perturbations of arbitrary size. Elongated perturbations near the axis of the filament should grow faster so prolate substructures and global collapse are favored, which is corroborated by most observations.

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## 1. Background: the ubiquity of filamentary structures

Filamentary structures are very common in astrophysics and are observed at various scales. On a cosmological scale, simulations show that matter is usually distributed along filaments, forming a cosmic web that connects galaxies to one another and provides the gas reservoir from which they grow and accrete (e.g., Springel et al. 2005, Dekel et al. 2009). The inner core of many of these cosmological filaments may be predominantly made of gas (Harford & Hamilton 2011). In the interstellar medium, observations show filamentary structures on much smaller scales. The formation of turbulence-driven filaments in the interstellar medium indeed seems to represent the first step towards core and star formation (André et al. 2010, Arzoumanian et al. 2011). Within a cosmic filament, matter can possibly contract and form galaxies, whereas an interstellar gas filament can clump into a series of bead-like structures which can then turn into stars.

Our goal is to obtain a dispersion relation for perturbations arising in an idealized filament and to compare its properties with available observations and simulations. The standard Jeans instability describes the collapse of a spherical gas cloud, but the cylindrical case is less straightforward and has not been fully investigated yet.

## 2. Local perturbations in a rotating filament

We consider an idealized, infinite, self-gravitating cylinder with pressure and density related by a barotropic equation of state. We neglect the role of magnetic fields for

simplicity, treat matter as an inviscid fluid and use hydrodynamics to obtain the linearized equations that govern the local perturbations (e.g., Mikhailovskii & Fridman 1973). We assume that there is no radial velocity in the undisturbed system and that all fluid particles share the same initial axial velocity, and further introduce axisymmetric perturbations of the generic form  $e^{-i\omega t} e^{ik_R R} e^{ik_z z}$ .

We first assume local perturbations: the typical scale of the perturbation is small compared to that of the unperturbed quantities, i.e.,  $k_R R_0 \gg 1$ , where  $R_0$  is the typical radius for the unperturbed distribution. This assumption is analogous to the Wentzel-Kramers-Brillouin approximation (WKB) used in quantum physics and leads to the following local dispersion relation (Freundlich et al. 2014a):

$$\omega^4 + \omega^2 (4\pi G \rho_0 - c_0^2 k^2 - \kappa^2) + \kappa^2 k_z^2 \left( c_0^2 - \frac{4\pi G \rho_0}{k^2} \right) = 0$$

where  $k = \sqrt{k_R^2 + k_z^2}$  corresponds to the total wavenumber and  $\kappa$  is the epicyclic frequency,  $\rho_0(R)$  and  $c_0(R)$  being respectively the initial density distribution and the effective sound speed. Rotation induces asymmetries but the boundary between the stable and unstable regimes is symmetrical in the phase plane  $(k_R, k_z)$ : the system is stable when  $k^2 \geq k_{\text{crit}}^2$  with  $k_{\text{crit}}^2 = 4\pi G \rho_0 / c_0^2$ , and unstable below, which corresponds to the standard Jeans criterion. When there is no rotation, the dispersion relation further reduces to the standard dispersion relation for collapsing spherical systems.

### 3. Global perturbations in a non-rotating filament

The local WKB assumption prevents the perturbations feeling the large-scale geometry of the system and thus leads to the standard spherical Jeans case when there is no rotation. Without this assumption and for non-rotating cylinders, the dispersion relation retains its complex terms and all modes are thus potentially unstable (Freundlich et al. 2014b). This dispersion relation yields that elongated perturbations near the axis of the filament grow faster, thus favoring elongated substructures. This is corroborated by observations, as most observations in the Taurus molecular cloud or in other molecular clouds favor prolate structures within interstellar filaments and tend to show that cores are elongated along the direction of the filaments (e.g., Curry 2002, Hartmann 2002).

Our model assumes an infinite and isolated filament, so it would be interesting to generalize our calculations to a cylinder of finite size and to take into account environmental effects and more complex velocity distributions. This work will be complemented by numerical studies of the formation and subsequent collapse of such idealized filaments.

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